Continuum Mechanics Quiz

Solutions

1.) Stresses, body forces, deacceleration and deconvection are the four components. A dynamic equation captures the conservation of momentum and relates the velocity vector to the stress tensor

for the x component of a Cartesian coordinate system:

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \rho g_x$$

2.) The continuity equation is strictly a kinematic constraint among the velocity gradients and gives no information regarding the stresses, and thus the type of fluid. The continuity equation for an incompressible fluid in Cartesian coordinates is

$$0 = \nabla \cdot \rho u$$

3.) Kinematics refers to the analysis and description of motion of an object through space. Kinematics differs from dynamics in that dynamics is interested in relating the motion of an object to the force that is causing that motion.

4.) Constitutive equations relate the components of the stress tensor to the kinematics by utilizing Δ (the rate of deformation tensor), and thus define the type of fluid that is being modeled.

$$T = -p\delta + \tau$$

p is representative of the normal stress component, τ is representative of the dynamic stress components

Note: δ is the unit tensor and is given by

$$\delta = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Continuum Mechanics Quiz - Continued

Solutions

5) The spatial coordinates for the displacement are:

$$x_1 = X_1 + 3X_2 - 4X_3$$
$$x_2 = 2X_1 + X_2 - X_3$$
$$x_3 = -X_1 + 4X_2 + X_3$$

The displaced position of particle A is given by:

$$x_1 = 1 + 3(0) - 4(3) = -11$$
$$x_2 = 2(1) + 0 - 3 = -1$$
$$x_3 = -1 + 4(0) + 3 = 2$$

The displaced position of particle B is given by:

$$x_1 = 3 + 3(6) - 4(6) = -3$$
$$x_2 = 2(3) + 6 - 6 = 6$$
$$x_3 = -3 + 4(6) + 6 = 27$$

The displaced position of the vector joining A and B is

$$\bar{V} = (-3-(-11))\hat{e}_1 + (6-(-1))\hat{e}_2 + (27-2)\hat{e}_3$$
$$\bar{V} = 8\hat{e}_1 + 7\hat{e}_2 + 25\hat{e}_3$$