Quiz 9 Polymer Properties 5/31/01

a) The Finger tensor for uniaxial extension is given by:

$$B_{\alpha\beta} = \begin{pmatrix} 1/\lambda & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & \lambda^2 \end{pmatrix}$$

Give the expression for the stress tensor in a bulk rubber and
From this give an expression for the tensile stress σ_{zz} in terms of λ.
How does this relate to the Mooney-Rivlin Plot?

b) For simple shear extension the Finger tensor is:

$$B_{\alpha\beta} = \begin{pmatrix} (\gamma^2 + 1) & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Write expressions for the shear stress, σ_{xy}, and
the first, $\sigma_{xx} - \sigma_{zz}$, and
the second, $\sigma_{xx} - \sigma_{yy,}$ normal stress differences.
Are these results consistent with Hookean behavior?

c) Shear flow can be described using the equation of state for a rubber (expression for stress from question "a") with a constant rate of shear strain (question "b") and applying the Boltzmann superposition position principle. The Boltzmann principle states that the present state of stress (or other property) is a result of the accumulated strains times a time dependent modulus (memory function).
Do you expect this approach to yield a first normal stress difference, $\sigma_{xx} - \sigma_{zz}$? Why?
Do you expect this approach to yield a second normal stress difference, $\sigma_{xx} - \sigma_{yy}$? Why?

d) The Stress-Optical Law states that $\Delta n = C_{opt} (\sigma_{zz} - \sigma_{xx})$ Doi/Edwards p. 222
i. **Does C_{opt} depend** on temperature?
ii. **Does C_{opt} depend** on molecular weight?
iii. **Does C_{opt} depend** on branching or cross link density?
iv. **Does C_{opt} depend** on polymer concentration in a concentrated solution?
v. **Does C_{opt} depend** on the relationship between stress and shear rate in flow?

e) The Stress-Optical Law indicates that stress and birefringence (Δn) have the same physical origin. Doi/Edwards p. 222
What is the physical origin that unites these two properties?

Answers: Quiz 9 Polymer Properties 5/31/01

a) $\sigma = \mathbf{G \bullet B} - P\,\mathbf{I}$

$\sigma_{zz} = \nu_c\, kT\lambda^2 - P$

$\sigma_{xx} = \sigma_{yy} = \nu_c\, kT/\lambda - P$

for $\sigma_{xx} = \sigma_{yy} = 0$, $P = \nu_c\, kT/\lambda$, and

$\sigma_{zz} = \nu_c\, kT(\lambda^2 - 1/\lambda)$

The latter equation is the basis of the Mooney-Rivlin Plot (Strobl p. 323) of reduced tensile stress, $\sigma/(\lambda - 1/\lambda^2)$, versus the inverse of strain, $1/\lambda$.

b) $\sigma_{xy} = (n_c/V)\, kT\, \gamma$

$\sigma_{xx} - \sigma_{zz} = (n_c/V)\, kT\, \gamma^2$

$\sigma_{yy} - \sigma_{zz} = 0$

The expression for the shear stress is Hook's Law for shear if $(n_c/V)\, kT$ is constant in strain. The first normal stress difference is not consistent with Hook's Law since Hookean behavior does not accommodate a normal stress difference. The second normal stress difference is consistent with Hook's law.

c) A first normal stress difference is expected since question b results in a first normal stress difference. A second normal stress difference is not expected since question b does not predict a second normal stress difference.

d) C_{opt} is linear in temperature

C_{opt} does not depend on molecular weight

C_{opt} does not depend on cross linking or branching

C_{opt} does depend on polymer concentration

C_{opt} does not depend on the relationship between stress and strain rate.

e) "Orientational ordering in the polymer segments" Doi/Edwards p. 222 The theory of polymer dynamics.