

# Lecture 19

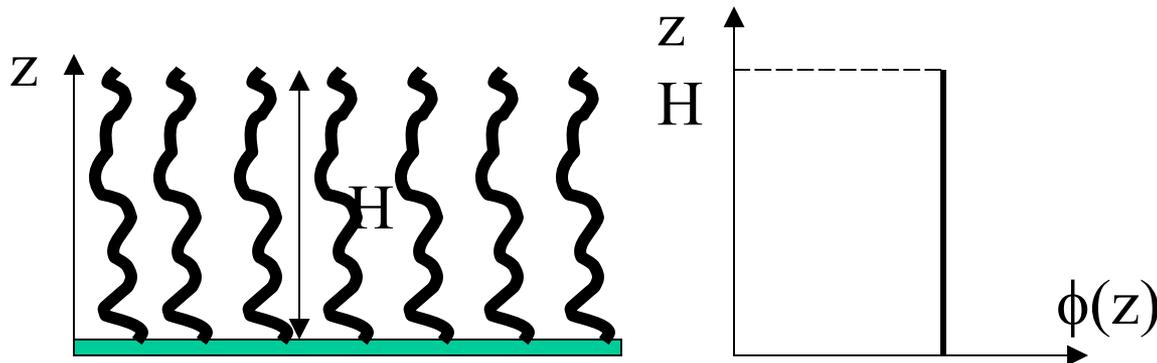
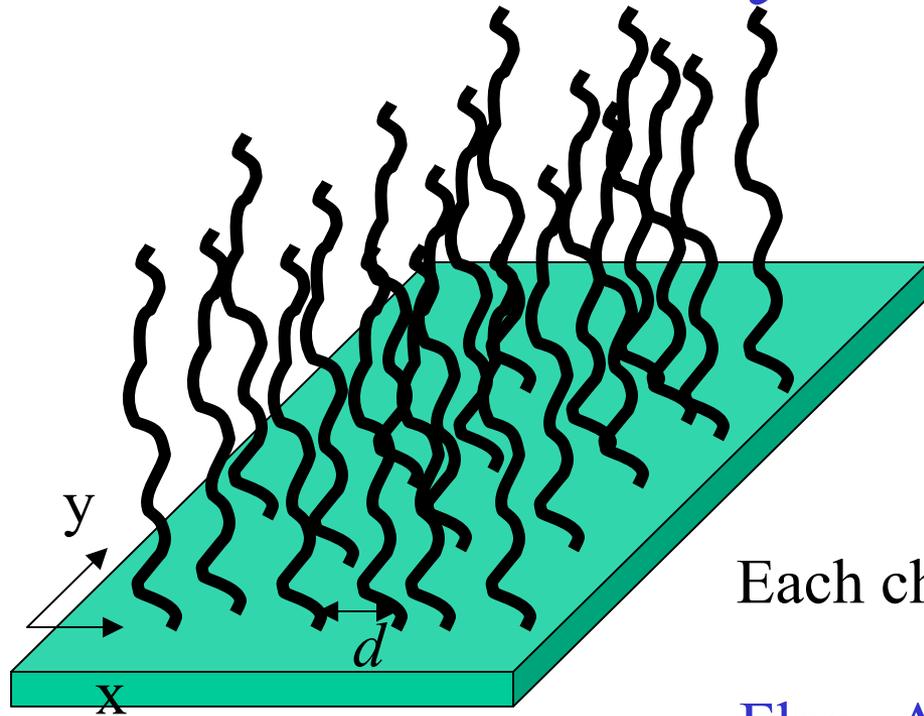
# Polymer Brushes

Consider a polymeric chains grafted to a surface by one end. The chains are separated by distance  $d$  from each other along  $x$  and  $y$  axes.

Each chain has a degree of polymerization  $N$ .

## Flory Approach

All free ends of grafted chains are located at the same distance  $H$  from the surface resulting in uniform density distribution.



# Polymer Brushes

## Flory Approach in $\theta$ -solvent

There are two contributions to the chain's free energy.

**Elastic energy** – associated with chain's conformational degrees of freedom

$$F_{elast} \approx \frac{S}{d^2} kT \frac{H^2}{b^2 N}$$

Total number of chains in the layer

Deformation of each chain.

**Interaction energy** which is due excluded volume interaction between monomers.

$$F_{int} \approx kT \frac{SH}{b^3} \phi^3 \quad \text{Three body interactions}$$

Where polymer volume fraction  $\phi$  is equal to  $\phi \approx \frac{S}{d^2} \frac{Nb^3}{SH} \approx \frac{Nb^3}{d^2 H}$

# Polymer Brushes

## Flory Approach in $\theta$ -solvent

The total free energy is

$$\frac{F}{kT} \approx \frac{S}{d^2} \left[ \frac{H^2}{b^2 N} + \frac{N^3 b^6}{H^2 d^4} \right]$$

The equilibrium thickness of the brush is obtained by minimizing the free energy with respect to brush height  $H$

$$\frac{\partial}{\partial H} \frac{F}{kT} \approx \frac{S}{d^2} \left[ 2 \frac{H}{b^2 N} - 2 \frac{N^3 b^6}{H^3 d^4} \right] = 0$$

$$H \approx \frac{b^2}{d} N$$

The chains in a brush are strongly stretched  $H \sim N$ , and thickness increases when the distance  $d$  between chains decreases.

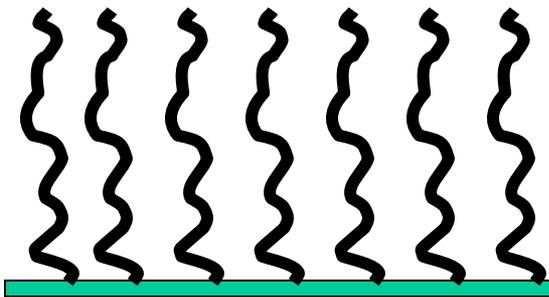
# Polymer Brushes

## Flory Approach in $\theta$ -solvent

Crossover to unperturbed chain regime occurs when the thickness of the layer is equal to ideal chain size  $bN^{1/2}$ .

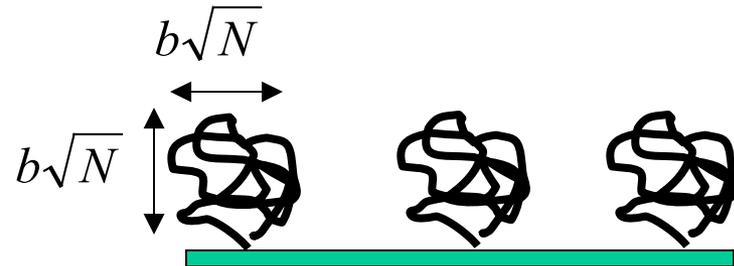
$$b\sqrt{N} \approx H \approx \frac{b^2}{d} N \Rightarrow d \approx b\sqrt{N}$$

$$d < b\sqrt{N}$$



Stretched chains

$$d > b\sqrt{N}$$

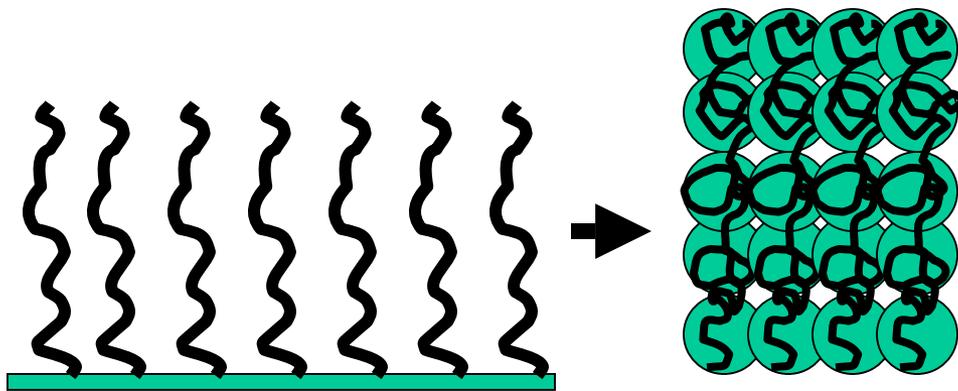


Ideal chains

# Polymer Brushes

## Scaling Approach in $\theta$ -solvent

In the scaling approach each chain is considered to be constraint within a tube of diameter  $d$  formed by surrounding chains



### Chain of blobs

Number of monomers in a blob

$$d \approx bg^{1/2} \Rightarrow g \approx d^2 / b^2$$

Brush thickness

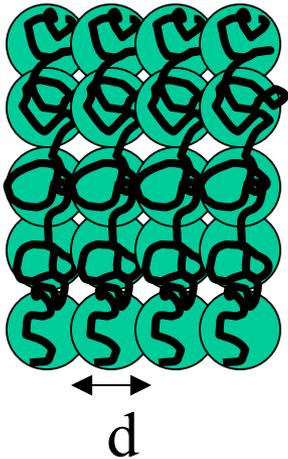
$$H \approx \frac{N}{g} d \approx \frac{Nb^2}{d}$$

Free energy of a chain in a brush

$$F_{ch} \approx kT \frac{N}{g} \approx kTN \frac{b^2}{d^2}$$

# Polymer Brushes

## Scaling Approach in Arbitrary Solvent



Number of monomers in a blob

$$d \approx bg^{\nu} \Rightarrow g \approx \left(\frac{d}{b}\right)^{1/\nu}$$

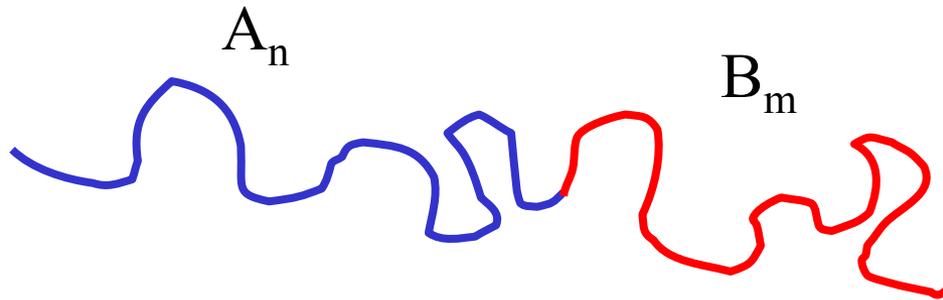
Brush thickness

$$H \approx \frac{N}{g}d \approx N \frac{b^{1/\nu}}{d^{1/\nu-1}}$$

Free energy of a chain in a brush

$$F_{ch} \approx kT \frac{N}{g} \approx kTN \left(\frac{b}{d}\right)^{1/\nu}$$

# Block Copolymers



Chain degree of polymerization  $N=m+n$

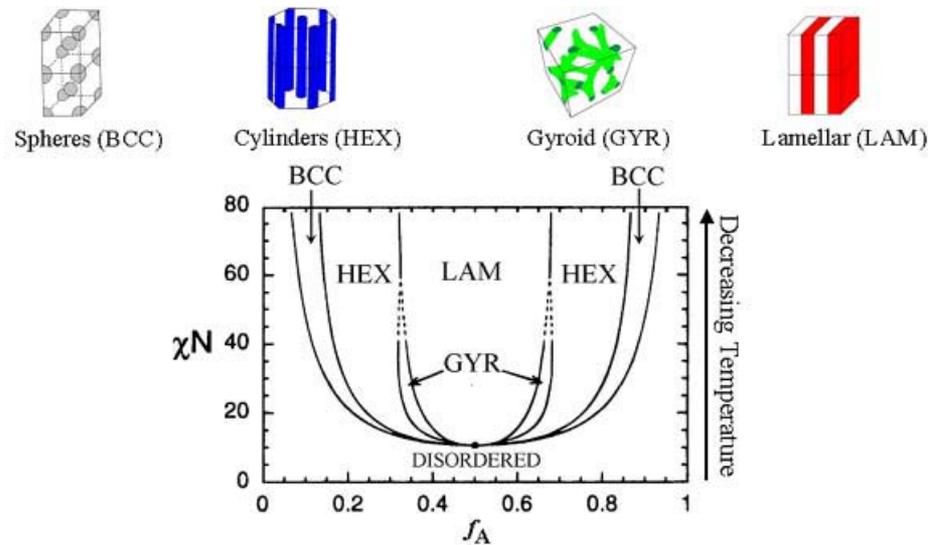
Chain composition  $f_A=n/(n+m)$

What will happen with melt of this polymers if the Flory-Huggins parameter  $\chi$  is large?

## **MICROPHASE SEPARATION**

Optimization of interaction between A and B blocks is achieved by redistributing monomers locally – forming domains rich with one component.

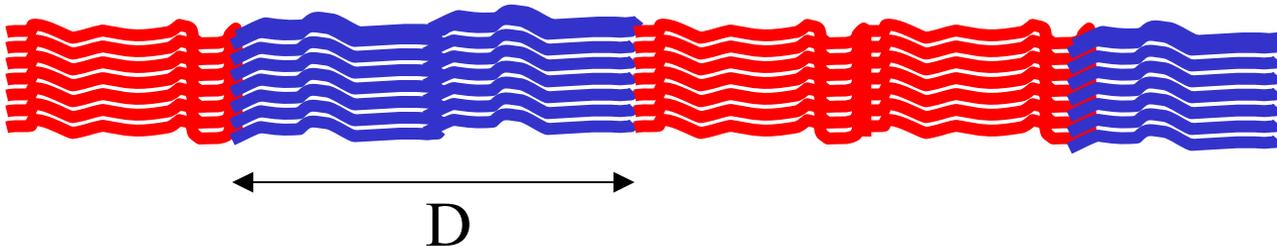
# Phase Diagram of Block Copolymers



Phase diagram of diblock copolymers:  $f_A$  polymer composition,  $\chi$  – Flory-Huggins interaction parameter,  $N$  - diblock degree of polymerization. Known equilibrium mesophases are S(pheres), C(ylinders), G(yroid), and L(amellae), as well as the disordered (DIS, homogeneous) at small interblock segregation strength ( $\chi N$ ). Diagram adapted from Matsen and Bates (1996).

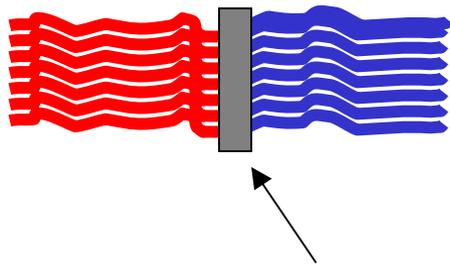
# Block Copolymers

Equilibrium size of the domains



Consider domains as brush-like layers of thickness  $D$  which grafting density should be optimized to minimize the chain free energy

Free energy of a chain in the domain



Interphase between  
A-rich and B-rich regions  
 $\gamma$ -surface energy

$$F \approx kT \frac{D^2}{b^2 n} + kT \frac{D^2}{b^2 m} + \gamma d^2$$

Elastic Energy

Interfacial Energy

In a melt polymer volume fraction is equal to 1.

$$1 = \phi = \frac{Nb^3}{Dd^2} \Rightarrow d^2 = \frac{Nb^3}{D}$$

# Block Copolymers

Using relation between distance between chains  $d$  and domain thickness  $D$  we can write

$$F \approx kT \frac{D^2}{b^2 n} + kT \frac{D^2}{b^2 m} + \gamma \frac{Nb^3}{D} \approx kT \frac{D^2}{b^2 N} + \gamma \frac{Nb^3}{D}$$

Minimizing the chain's free energy with respect to domain size  $D$  one has

$$\frac{\partial F}{\partial D} \approx 2kT \frac{D}{b^2 N} - \gamma \frac{Nb^3}{D^2} \quad D \approx bN^{2/3} \left( \frac{\gamma b^2}{kT} \right)^{1/3}$$

As in the case of polymeric brush chains are strongly stretched.

The interfacial energy  $\gamma$  is related to the Flory-Huggins parameter  $\chi$

$$\gamma \approx \frac{kT}{b^2} \chi^{1/2}$$

# Polymer Adsorption

Correlation length in semidilute solution

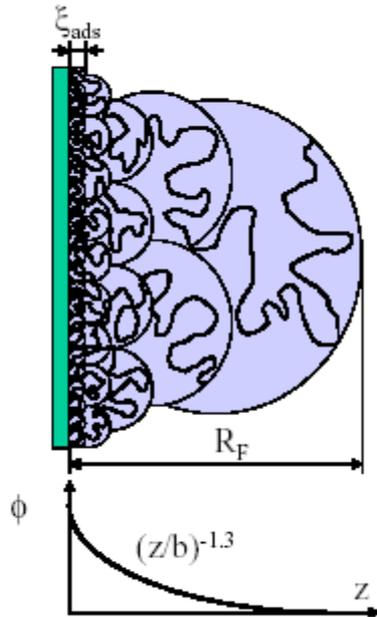
$$\xi \approx b \phi^{-\nu / (3\nu - 1)}$$

de Gennes assumption:  $\xi = z$

**de Gennes self-similar carpet**

Polymer density profile inside adsorbed layer

$$\phi(z) \approx \left( \xi(z) / b \right)^{-(3\nu - 1) / \nu} \approx \left( z / b \right)^{-(3\nu - 1) / \nu}$$



Polymer surface coverage

$$\Gamma \approx \int \frac{\phi(z)}{b^3} dz \approx b^{-3} \int_{\xi_{ads}}^{R_F} (z/b)^{-(3\nu - 1) / \nu} dz \approx b^{-2} \left( \frac{b}{\xi_{ads}} \right)^{(2\nu - 1) / \nu}$$

Adsorption blob:  $\xi_{ads} \approx b \delta^{-\nu / (1 - \nu)}$

# Summary of Polymer Solutions

