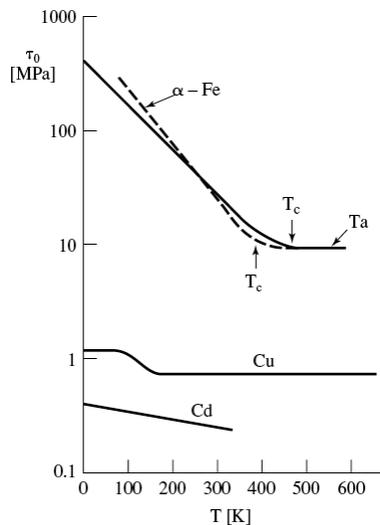


Quiz 2 Properties of Materials CME 300
October 5, 2011

- a) Give the tensoral matrix for stress and strain showing shear (square) and tensile (circle) components (use double bar hats for tensors)? Write an equation that defines stress and strain in terms of vectors (use single bar hats for vectors). Show in a drawing one tensile and one shear component of the stress and strain matrices (show the force, area etc vectors for these components).
- b) In Chemical Engineering you are often interested in flow of a fluid in a pipe that follows Poiseuille's law. Is flow in a pipe a tensile or a shear deformation? Show the components of the stress and strain rate (velocity and direction of change of velocity) in a sketch related to pipe flow to support your proposition.
- c) For an FCC metal like Cu sketch the stress-strain plot showing the Hookean elastic region, the yield stress and yield strain. What happens if the stress is released before the yield point? What happens if the stress is released after the yield point? (Sketch the recovery curves for these two cases using a dashed line.)
- d) The following is a plot of the critical resolved shear stress versus absolute temperature that was given in class. Write an equation for the behavior of α -Fe and Ta ($\tau_0 = f(T)$) in the linear region and explain the terms in this equation including a definition of τ_0 .



- e) The rate of crystallization for metals is many orders of magnitude faster than the rate for polymer crystallization. Explain why the rate might be different (give at least two possible reasons).

Answers Quiz 2 CME 300 October 5, 2011

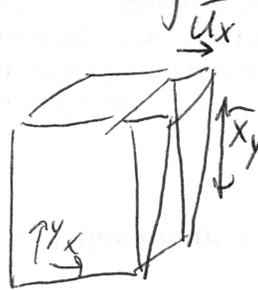
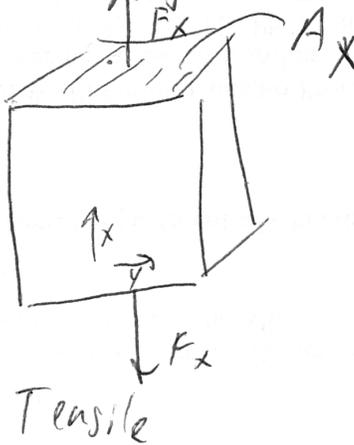
a)

$$\bar{\sigma} = \frac{F_x}{A_x} \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

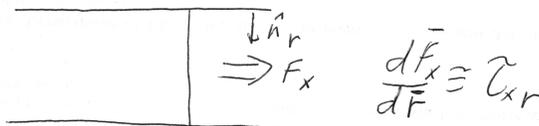
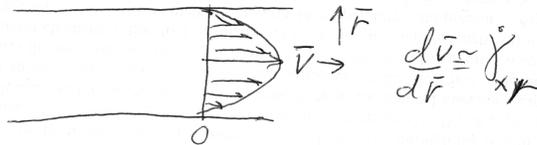
$$\bar{\epsilon} = \frac{u}{x} \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}$$

$$\bar{\sigma}_{ij} = \frac{\partial \bar{F}_i}{\partial \bar{X}_j}$$

$$\bar{\epsilon}_{ij} = \frac{\partial \bar{u}_i}{\partial \bar{X}_j}$$



b) Flow in a pipe is shear flow. The fluid reaches a maximum velocity at the center of the pipe and has 0 velocity at the walls (wall stick assumption). The gradient across the pipe is non-linear since the diameter and differential volume of fluid changes with the radius.



c)

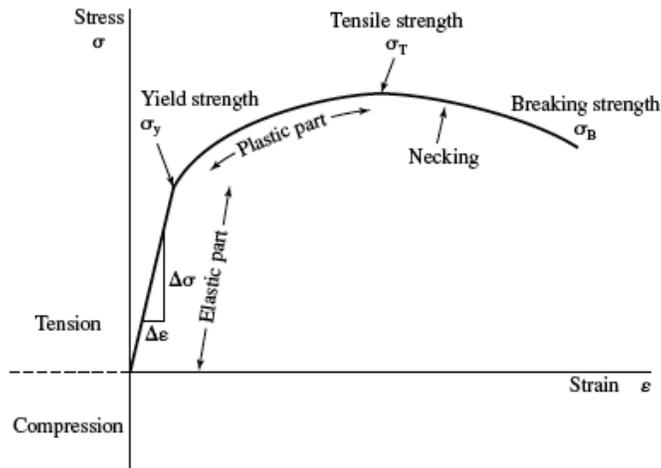


FIGURE 2.4. Schematic representation of a stress-strain diagram for a ductile material. For actual values of σ_y and σ_T , see Table 2.1 and Figure 2.5.

If stress is released before the yield point the relaxation curve will follow the stress strain curve exactly. If stress is released after the yield point there is hysteresis and a linear relaxation occurs to a point above 0 strain.

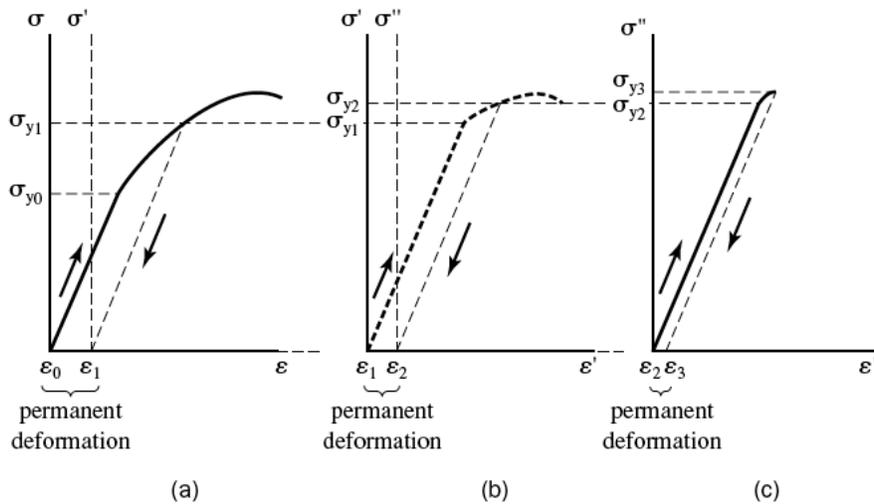


FIGURE 2.8. Increase of yield strength (and reduction of ductility) by repeated plastic deformation. (a) Sample is moderately stressed until some plastic deformation has occurred, and then it is unloaded, which yields permanent deformation. (b) The sample is subsequently additionally permanently deformed. Note that the coordinate system has shifted after unloading from ϵ_0 to ϵ_1 . (c) Limit of plastic deformation is reached after renewed stressing.

d) τ_0 is the critical resolved shear stress is the shear stress needed to initiate plastic deformation for a crystal. In the figure $\log(\tau_0)$ is linear in temperature up to the critical temperature T_c . For the linear region,

$$\tau_0 = \exp(-K_1 T + K_2)$$

beyond this region a more complicated function is needed. K_2 is the natural log of the critical shear stress at absolute 0, K_1 is the rate of change of the log of critical shear stress as a function

of temperature. K_1 is probably related to $k_B/\Delta E$, where k_B is the Boltzmann constant and ΔE is the barrier energy for slip in the BCC system. (Simply adding a constant to the equation in order to mimic the plateau at high T doesn't work with this data.)

e) The viscosity of polymers is much higher than metals, for instance the viscosity of copper at the melting point is about 0.04 poise while the viscosity of polyethylene (zero shear viscosity) ranges from 10^4 to 10^8 poise depending on molecular weight. Higher viscosity makes flow and motion of the molecules much slower since the Einstein-Stokes equation indicates that the diffusion coefficient is inversely related to the viscosity and higher D means slower motion ($D \sim kT/(\eta R)$).

Polymer crystallization requires a sequence of steps that are not necessary for metal crystallization. For metals a metal atom simply diffuses to the crystal and attaches with a release of enthalpy. For polymers the chain must become disentangled from the melt, form a helix, approach the crystal with the correct orientation, bind with the crystal, chain fold to add the next helical segment. This takes time so the rate is much slower.

The enthalpy of crystallization is not related to this, though you might think that a larger enthalpy of crystallization could lead to faster crystallization. In fact, the enthalpy of crystallization for polyethylene is 286.7 J/g while for copper it is 176 J/g so about the same enthalpy of crystallization