CME 300 Properties of Materials

ANSWERS: Homework 9 November 26, 2011

18.7 How does the electron structure of an isolated atom differ from that of a solid material?

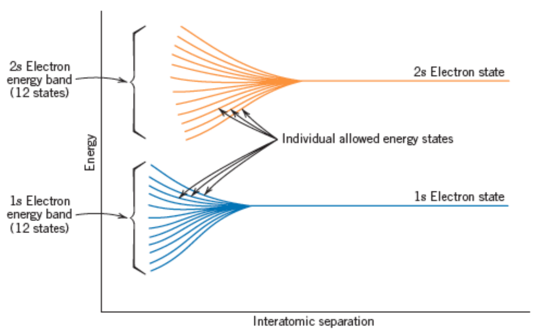
As atoms approach each other in the solid state the quantized energy states:

Table 2.1 The Number of Available Electron States in Some of the Electron Shells and Subshells

Principal Quantum Number n	*Shell Designation*	*Subshells*	*Number of States*	*Number of Electrons Per Subshell*	*Number of Electrons Per Shell*
1	*K*	*s*	1	2	2
2	*L*	*s*	1	2	8
		p	3	6	
3	*M*	*s*	1	2	18
		p	3	6	
		d	5	10	
4	*N*	*s*	1	2	32
		p	3	6	
		d	5	10	
		f	7	14	

are split. This splitting is associated with the wave nature of the electron in an energy level. Waves can diffract from the valence electrons of other atoms in the solid. The splitting becomes more pronounced the closer atoms are from each other and the number of splittings is related to the number of atoms. In a solid the splitting lead to a continuous band of electron energies up to the Fermi Energy for the highest occupied energy of the valance band.

Figure 18.2 Schematic plot of electron energy versus interatomic separation for an aggregate of 12 atoms ($N = 12$). Upon close approach, each of the 1*s* and 2*s* atomic states splits to form an electron energy band consisting of 12 states.



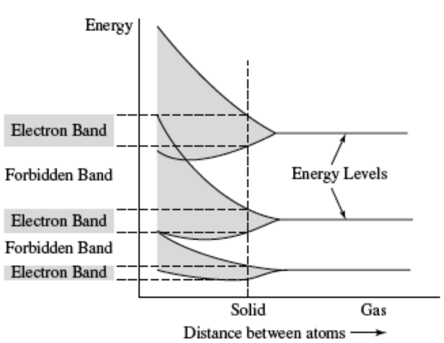


FIGURE 11.5. Schematic representation of energy levels (as for isolated atoms) and widening of these levels into energy bands with decreasing distance between atoms. Energy bands for a specific case are shown at the left of the diagram.

18.8 In terms of electron energy band structure, discuss reasons for the difference in electrical conductivity between metals, semiconductors, and insulators.

The band structure is composed of filled states at the lowest energies, the valance band at a moderate energy where the highest occupied energy levels exist and a generally empty conduction band (for semiconductors and insulators). The size of the forbidden gap between the valance and the conduction bands decides if a material is a conductor, semiconductor or insulator.

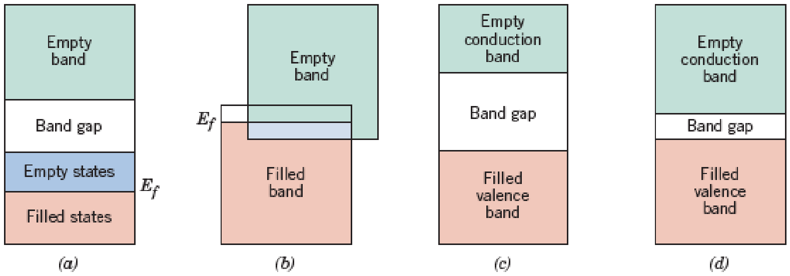


Figure 18.4 The various possible electron band structures in solids at 0 K. (*a*) The electron band structure found in metals such as copper, in which there are available electron states above and adjacent to filled states, in the same band. (*b*) The electron band structure of metals such as magnesium, wherein there is an overlap of filled and empty outer bands. (*c*) The electron band structure characteristic of insulators; the filled valence band is separated from the empty conduction band by a relatively large band gap (>2 eV). (*d*) The electron band structure found in the semiconductors, which is the same as for insulators except that the band gap is relatively narrow (<2 eV).

For conductors the valance band and the conduction band overlap. For insulators there is a large band gap between the valance and the conduction bands. For semiconductors there is a small band gap between the valance and conduction energy bands.

18.19 For intrinsic semiconductors, the intrinsic carrier concentration n_i depends on temperature as follows:

$$n_i \propto \exp\left(-\frac{E_g}{2kT}\right) \qquad (18.35a)$$

or taking natural logarithms,

$$\ln n_i \propto -\frac{E_g}{2kT} \qquad (18.35b)$$

Thus, a plot of $\ln n_i$ versus $1/T$ (K)$^{-1}$ should be linear and yield a slope of $-E_g/2k$. Using this information and the data presented in Figure 18.16, determine the band gap energies for silicon and germanium, and compare these values with those given in Table 18.3.

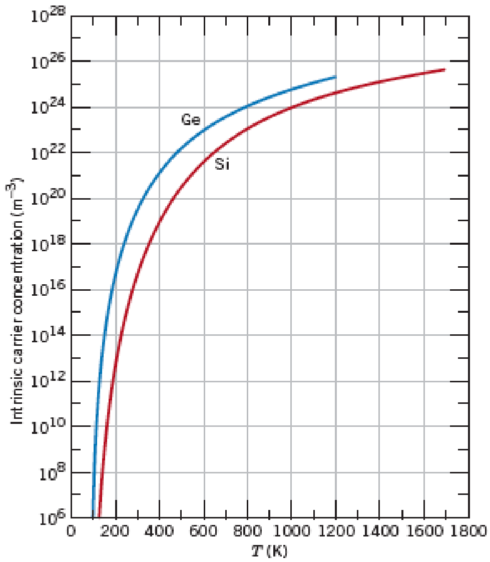
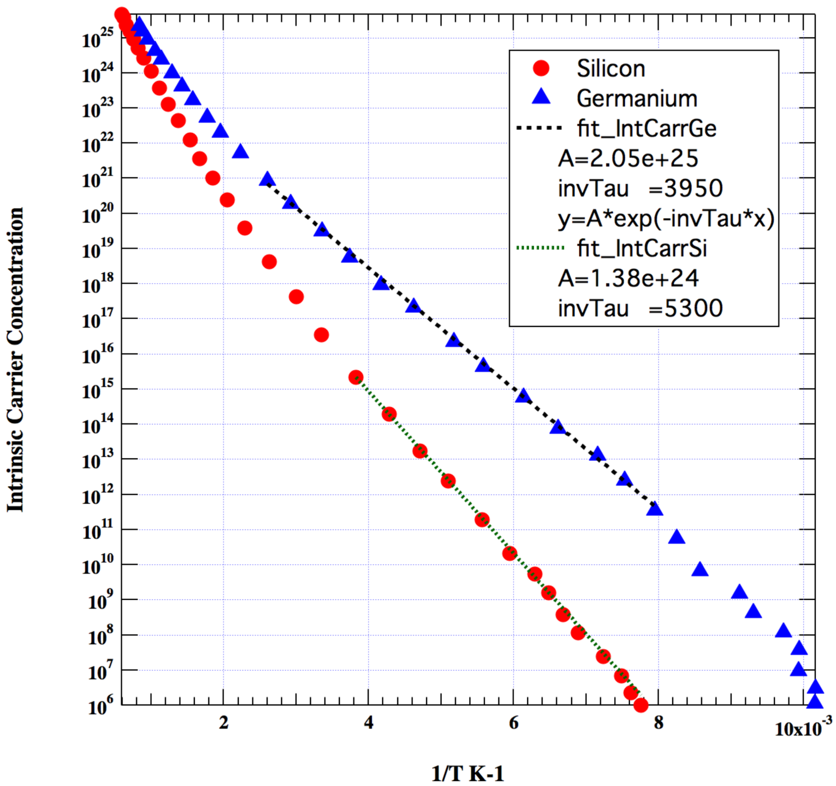


Figure 18.16 Intrinsic carrier concentration (logarithmic scale) as a function of temperature for germanium and silicon. (From C. D. Thurmond, "The Standard Thermodynamic Functions for the Formation of Electrons and Holes in Ge, Si, GaAs, and GaP," *Journal of The Electrochemical Society*, **122**, [8], 1139 (1975). Reprinted by permission of The Electrochemical Society, Inc.)

Table 18.3 Band Gap Energies, Electron and Hole Mobilities, and Intrinsic Electrical Conductivities at Room Temperature for Semiconducting Materials

Material	*Band Gap (eV)*	*Electrical Conductivity* $[(\Omega\text{-}m)^{-1}]$	*Electron Mobility* $(m^2/V\text{-}s)$	*Hole Mobility* $(m^2/V\text{-}s)$
		Elemental		
Si	1.11	4×10^{-4}	0.14	0.05
Ge	0.67	2.2	0.38	0.18
		III–V Compounds		
GaP	2.25	—	0.03	0.015
GaAs	1.42	10^{-6}	0.85	0.04
InSb	0.17	2×10^4	7.7	0.07
		II–VI Compounds		
CdS	2.40	—	0.03	—
ZnTe	2.26	—	0.03	0.01

Data was digitized using "PlotDigitizer" shareware and replotted as log carrier conc versus 1/T. This was fit with the exponential function given in the question to yield E/2k.



E_{Si} = 2*7200 * 8.62 e-5 eV/K = 1.2 eV (Table value 1.1 eV)
E_{Ge} = 2* 7200 * 8.62 e-5 eV/K = 0.914 eV (Table value 0.67 eV)

18.24 Define the following terms as they pertain to semiconducting materials: intrinsic, extrinsic, compound, elemental. Now provide an example of each.

Intrinsic means that the charge carriers are not from dopants so there are equal numbers of holes and electrons as charge carriers. This is an undoped semiconductor such as intrinsic Si or intrinsic Ge. Extrinsic semiconductors are doped materials with either electrons or holes as the majority carriers such a P doped Si or Al doped Si. A compound semiconductor is a semiconductor composed of different atoms such as GaAs from group III an group V. An elemental semiconductor is a semiconductor composed of a single element such as Si or Ge.

18.26 (a) In your own words, explain how donor impurities in semiconductors give rise to free electrons in numbers in excess of those generated by valence band–conduction band excitations. **(b)** Also explain how acceptor impurities give rise to holes in numbers in excess of those generated by valence band–conduction band excitations.

For each group V element (P) added to a group IV semiconductor (Si) you create one extra electron that can act as a charge carrier. Similarly, for each group III element (Al) added to a group IV semiconductor (Si) an electron hole is created that can act as a charge carrier.

18.28 Will each of the following elements act as a donor or an acceptor when added to the indicated semiconducting material? Assume that the impurity elements are substitutional.

Impurity	*Semiconductor*
N	Si
B	Ge
S	InSb
In	CdS
As	ZnTe

N (group V) to Si (group IV) so N is a donor of electrons.
B (group III) to Ge (group IV) so B is an acceptor of electrons (makes holes).
S (group VI) to InSb (groups III and V) so S is a donor of electrons.
In (group III) to CdS (groups II and VI) so In is an acceptor of electrons (makes holes).
As (group V) to ZnTe (groups II and VI) so As is a donor of electrons.

18.36 Compare the temperature dependence of the conductivity for metals and intrinsic semiconductors. Briefly explain the difference in behavior.
and doped (extrinsic) semiconductors.

The conductivity of metals decreases linearly with absolute temperature since the path of electrons in a metal becomes longer as an increase in temperature makes the diffusion path longer and this increases the number of collisions with other atoms leading to a loss following the Drude model.

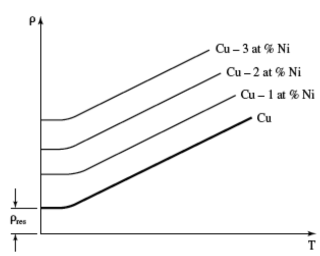
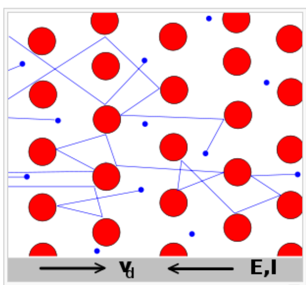


FIGURE 11.3. Schematic representation of the temperature dependence of the resistivity of copper and various copper-nickel alloys. ρ_{res} is the residual resistivity.



Drude Model electrons (shown here in blue) constantly bounce between heavier, stationary crystal ions (shown in red).

The conductivity of a semiconductor increases exponentially with temperature following an Arrhenius function.

$$N_e = 4.84 \times 10^{15}\, T^{3/2} \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right],$$

since the number of charge carriers increases as electrons can overcome the band gap through thermal motion. This process leads to one hole conductor in the valance band for every electron conductor in the conduction band.

18.41 Some hypothetical metal is known to have an electrical resistivity of 3.3×10^{-8} (Ω-m). Through a specimen of this metal 15 mm thick is passed a current of 25 A; when a magnetic field of 0.95 tesla is simultaneously imposed in a direction perpendicular to that of the current, a Hall voltage of -2.4×10^{-7} V is measured. Compute **(a)** the electron mobility for this metal, and **(b)** the number of free electrons per cubic meter.

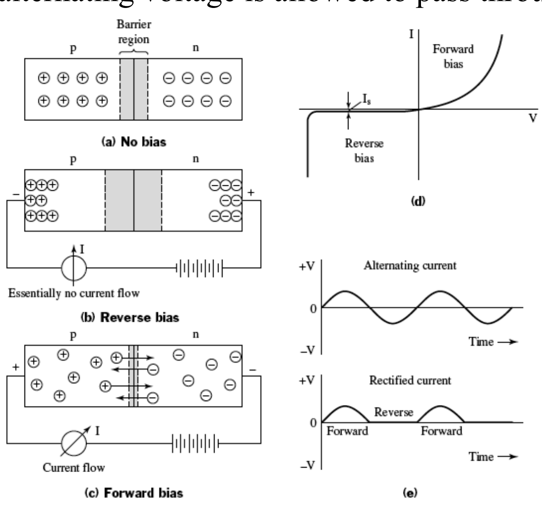
$\rho = 3.3 \times 10^{-8}$ Ωm. 15 mm = d. I = 25A. B = 0.95 Tesla. V = -2.4×10^{-7} V

R_H = Vd/IB = -2.4×10^{-7} V * 0.015 m/(25A 0.95 Tesla) = -1.52 x 10-10 Ωm/Tesla
$\mu_e = |R_H|\,\sigma = 1.52 \times 10^{-10}/3.3 \times 10^{-8}$ Ωm = 0.0046 Tesla-1=0.0046 $m^2/(Vs)$
$\sigma = 1/\rho$
n = $1/(R_H\, e)$ = 1/(1.52 x 10-10 Ωm /(Tesla) 1.602 x 10-19 Coulomb per electron) = 4.11×10^{28} electrons Tesla/(Coulomb Ωm) = 4.11×10^{28} electrons (V s/m^2)/((A s) (Ωm)) = 4.11×10^{28} electrons (V s/m^2)/(V s m) = 4.11×10^{28} electrons/m^3

18.43 Briefly describe electron and hole motions in a *p–n* junction for forward and reverse biases; then explain how these lead to rectification.

At the interface where p and n meet there is a flux of electrons from n to p that leads to a negative charge on the p side and a positive charge on the n side. Forward bias from p to n drives electrons to the positive layer of the n doped layer which has a low or no barrier. The reverse process (reverse bias) drives electrons from n to the negatively charged p interfacial layer so there is a barrier to this and the current will not flow until a breakdown voltage is reached.

This situation leads to rectification of an AC current since only the forward bias part of the alternating voltage is allowed to pass through the diode. This is shown in the figure below.



18.45 What are the two functions that a transistor may perform in an electronic circuit?
A transistor can function as a switch or as an amplifier.

Amplifier:

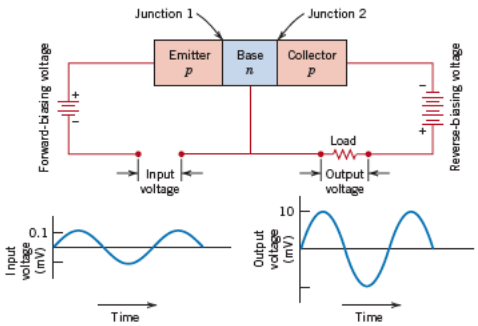
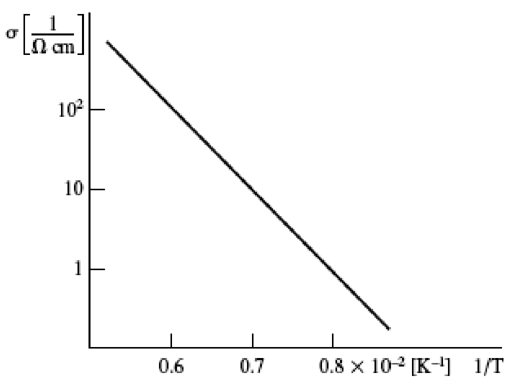


Figure 18.24 Schematic diagram of a *p–n–p* junction transistor and its associated circuitry, including input and output voltage–time characteristics showing voltage amplification. (Adapted from A. G. Guy, *Essentials of Materials Science*, McGraw-Hill Book Company, New York, 1976.)

18.46 Cite the differences in operation and application for junction transistors and MOSFETs.

A bipolar junction transistor is controlled by a small current. The current is amplified in the transistor. This is more amenable for use as an amplifier though it can be used as a switch. A MOSFET is controlled by a small voltage (typically ± 5 V) so that it is more amenable for use as a digital switch but it can be used to amplify a voltage signal.

11.7. In the figure below, σ is plotted as a function of the reciprocal temperature for an intrinsic semiconductor. Calculate the gap energy. (Hint: Combine (11.12) and (11.15) and take the ln from the resulting equation assuming $N_e \equiv N_h$ · Why?).)



$$\sigma = N_e \mu_e e + N_h \mu_h e,$$

$$N_e = 4.84 \times 10^{15}\, T^{3/2} \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right],$$

$$\sigma = 4.84x10^{15} e T^{3/2} \exp\left(-\frac{E_g}{2kT}\right)(\mu_e + \mu_h)$$

$$\ln\left(\frac{\sigma}{(\mu_e + \mu_h) 4.84x10^{15} e T^{3/2}}\right) = \left(-\frac{E_g}{2kT}\right) = \ln\sigma - \ln\left((\mu_e + \mu_h) 4.84x10^{15} e T^{3/2}\right)$$

Since the plot is linear we can assume that the second term in the last equality is small for this range of temperature. 1/T ~ 0.63 x 10-2 K-1 at s = 100/ohm cm. 1/T ~ 0.8 x 10-2 K-1 at s = 1 /ohm cm. So the slope is (ln100-ln1)/(0.63 – 0.8) x 10-2 K = -2700 K
Eg = 2*8.62 e-5 eV/K*2700 = 0.47 eV.
(There seems to be a problem with the units in this but the end result is in the correct range.)

1) Define the Drude Model, Quantum Mechanics Model, Density of States, Fermi Energy, Extrinsic Semiconductor, Intrinsic Semiconductor, Indirect Band Gap Semiconductor, Direct Band Gap Semiconductor, Homo-Junction, Hetero-Junction.

Drude Model describes conduction in a metal and the decrease in conductivity with temperature due to increase in the motion of the electrons in the lattice and the increased collision frequency with defects and atoms. The Drude Model is a classical model in that it does not depend on quantum mechanics.

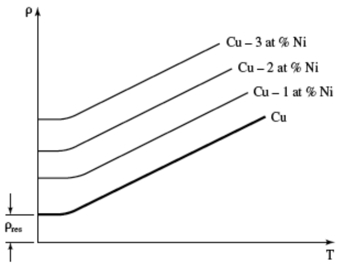


FIGURE 11.3. Schematic representation of the temperature dependence of the resistivity of copper and various copper-nickel alloys. ρ_{res} is the residual resistivity.

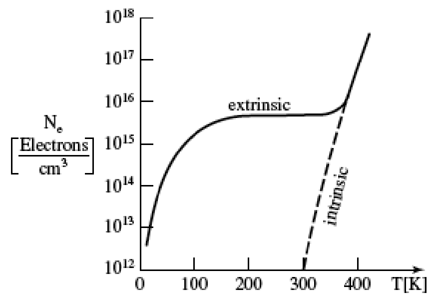
The quantum mechanics model is needed to describe conduction in semiconductors and describes conduction in metals according to a conduction band as discussed above. Conductivity in a metal is proportional to the number of Fermi level electrons in the quantum mechanics model.

The density of states describes the distribution of electrons in energy states within a band, particularly the valance band. The highest energy electrons in this distribution are in the Fermi level.

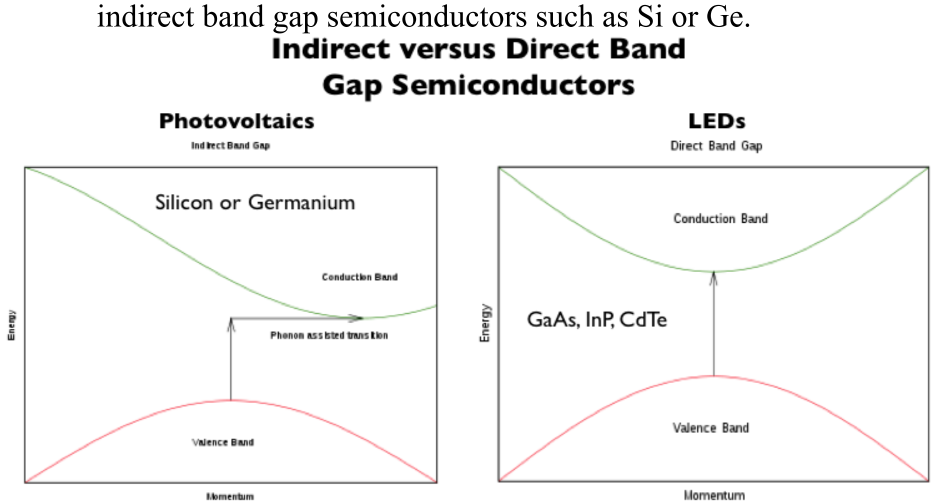
An intrinsic semiconductor is an undoped semiconductor that shows an exponential increase in conductivity with temperature as Fermi level electrons are thermally excited to the conduction band.

Extrinsic semiconductors are doped semiconductors where the conductivity depends on the level of doping and not on the temperature.

FIGURE 11.13. Schematic representation of the number of electrons per cubic centimeter in the conduction band as a function of temperature for extrinsic semiconductors, assuming low doping.



An indirect band gap semi conductor is a semiconductor that requires phonons (lattice vibrations with the generation of heat) to transfer an electron from the valance band to the conduction band. A direct band gap semiconductor does not involve a phonon in this transfer so the energy is given off as light rather than heat. LEDs and Laser Diodes require direct band gap semiconductors such as GaAs or CdTe and can not be made with indirect band gap semiconductors such as Si or Ge.



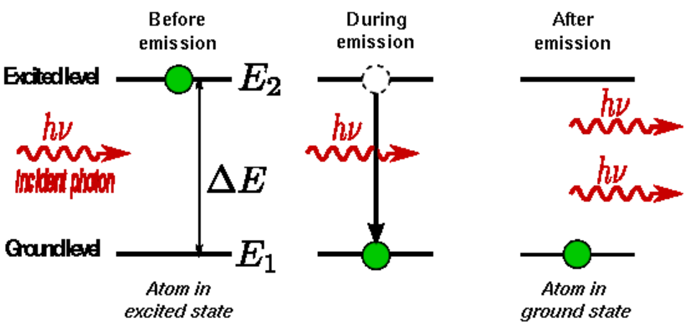
Energy vs. crystal momentum for a semiconductor with an indirect band gap, showing that an electron cannot shift from the lowest-energy state in the conduction band (green) to the highest-energy state in the valence band (red) without a change in momentum. Here, almost all of the energy comes from a photon (vertical arrow), while almost all of the momentum comes from a phonon (horizontal arrow).

Energy vs. crystal momentum for a semiconductor with a direct band gap, showing that an electron can shift from the lowest-energy state in the conduction band (green) to the highest-energy state in the valence band (red) without a change in crystal momentum. Depicted is a transition in which a photon excites an electron from the valence band to the conduction band.

A homojunction is a junction with a single band gap between the two materials such as a pn junction in Si or Ge. A heterojunction involves two different band gaps in the two materials such as in nanoparticles that are joined in crystallization. Heterojunctions can be tuned to enhance the interfacial region where charge transfer occurs in a diode.

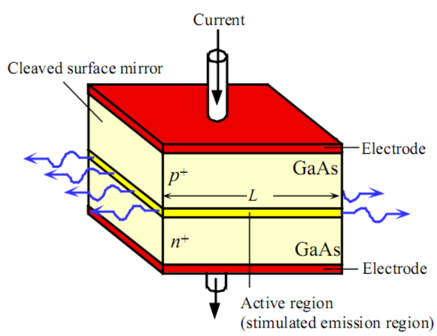
2) How does a Laser Diode function?

You need a direct band gap material so that the electrical energy can be directly converted to a photon rather than being mediated by a phonon. The laser diode functions similar to an LED except that the laser has an optical cavity with mirrors on the ends and a wave guide between so that the emitted photons can excite more emission at the same wavelength, phase and direction. This optical emission is called lasing.



$$E_2 - E_1 = \Delta E = h\nu$$

An adequate forward bias is required to inject carriers across the junction to initiate population inversion The process is called ***injection pumping.***



A schematic illustration of a GaAs homojunction laser diode. The cleaved surfaces act as reflecting mirrors.

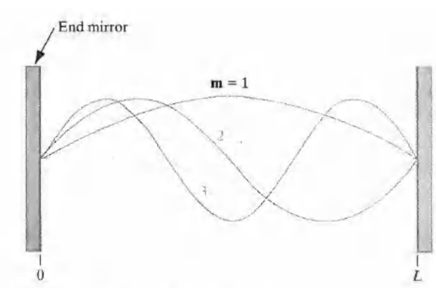


Figure 8–17
Resonant modes within a laser cavity.