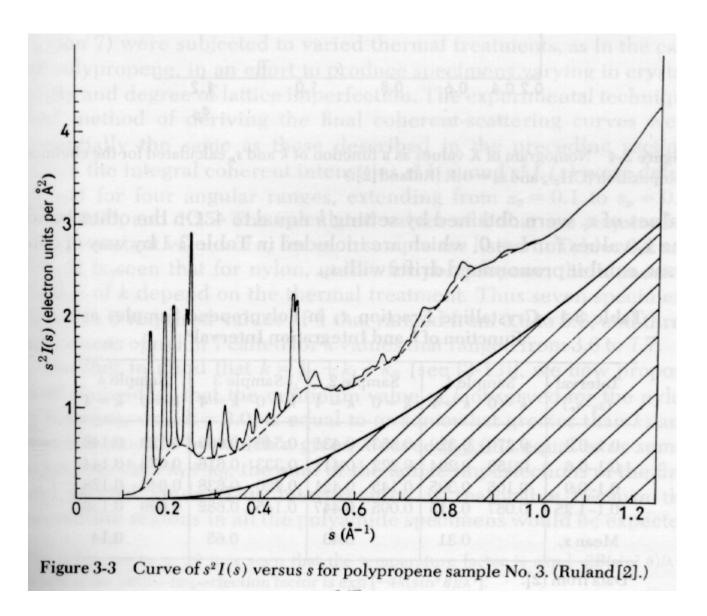
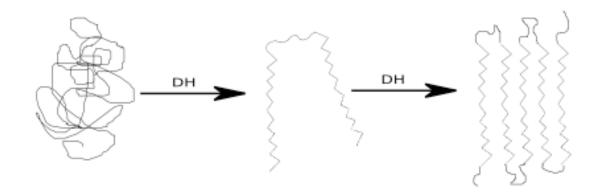


Semi-Crystalline
Helical Structure
Chain Folding
Semi-Crystalline Fibrillar Growth
Spherulitic, shish-kabob, epitaxial surface nucleation,
Crystalline Orientation
Lamellar Orientation
Macroscopic Orientation





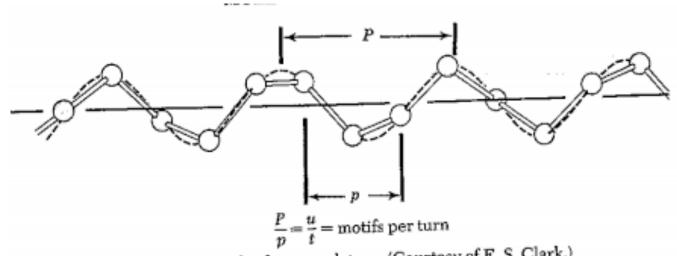


Figure 6-1 Helical nomenclature. (Courtesy of E. S. Clark.)

- p = Z-axis projection of the distance between consecutive equivalent points on the helix;
- u = number of points, or motifs, on the helix corresponding to the period c (an integer);
- t = number of turns of the helix in the identity period c (an integer);
- $\Delta \Phi = {
 m projection}$ on a plane perpendicular to Z of the central angle defined by two successive equivalent points on the helix;
 - $\gamma = a \text{ positive integer} > 0;$
 - $\epsilon = a \text{ positive integer} \ge 0.$

... Jissussed in detail

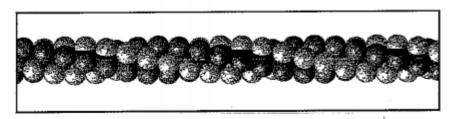


Fig. 2.3. PTFE in the crystalline state. The conformation corresponds to a 13/6helix

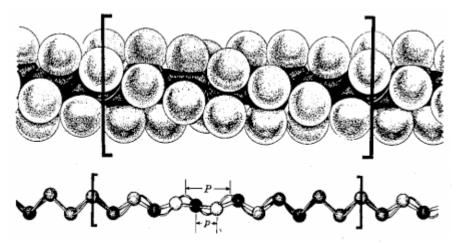


Figure 6-9 Mölecular conformation of polytetrafluoroethylene: top-repeat unit of one molecule; bottom-carbon skeleton showing 13/6 helix. (Clark and Muus [27].)

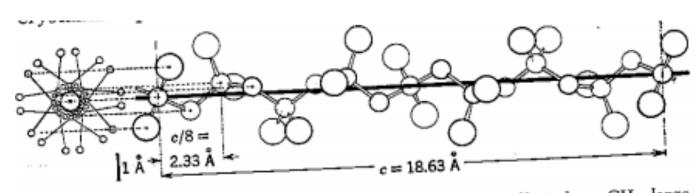


Figure 6-8 The 8/5 helix of the polyisobutene molecule. Small circles = CH₂; large circles = CH₃. (Liquori [25].)

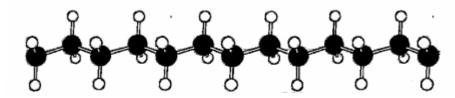


Fig. 2.1. The steric structure of PE. Rotations about the C-C bonds result in a change in the conformation

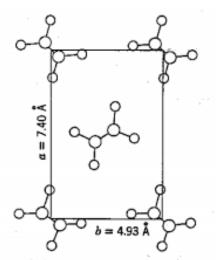


Figure 6-40 Arrangement of the molecules of polyethylene in the c-projection. (Courtesy of E. S. Clark.)

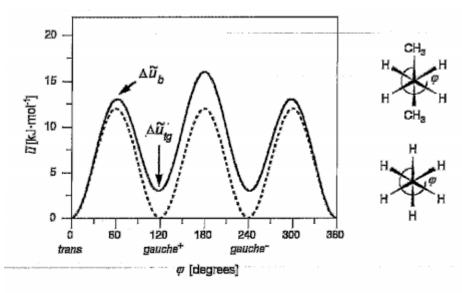


Fig. 2.2. Potential energies associated with the rotation of the central C-C bond for ethane (broken line) and butane (continuous line). The sketches show the two molecules in views along the C-C bond

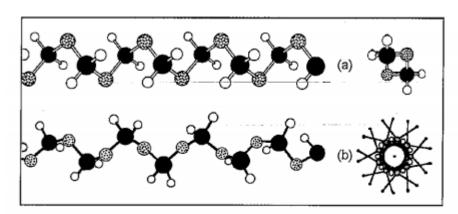


Fig. 2.4. Two different helices formed by POM: 2/1-helix (a) and 9/5-helix (b).
Side views (left) and views along the helix axis (right)

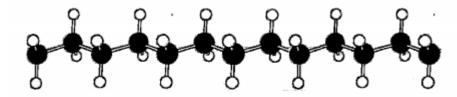


Fig. 2.1. The steric structure of PE. Rotations about the C-C bonds result in a change in the conformation

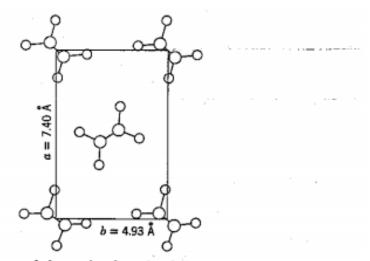


Figure 6-40 Arrangement of the molecules of polyethylene in the c-projection. (Courtesy of E. S. Clark.)

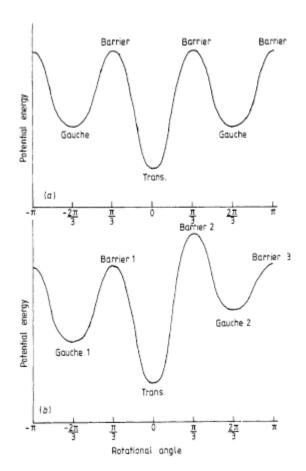


Figure 1. Schematic potential energy diagrams of rotation for (a) polyethylene and (b) a vinyl polymer.

Paul Phillips 1990 Review

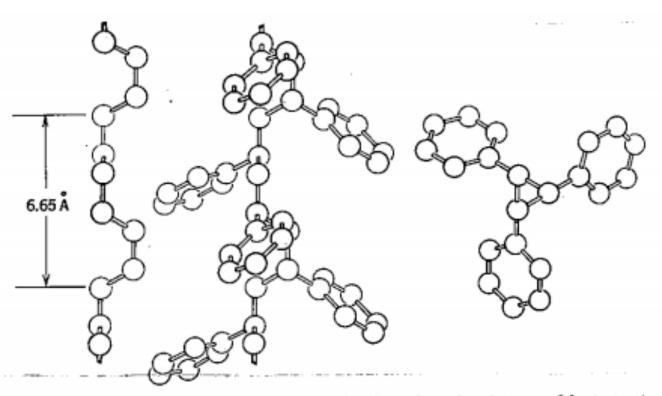


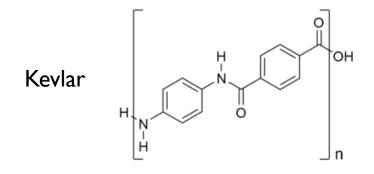
Figure 6-7 Left: the chain $(TG)_3$. Center and right: side and end views of the isotactic polystyrene molecule. (Bunn and Howells [17].)

P J Phillips

(a)

$$B \left[\begin{array}{c} S \\ \\ \\ \end{array} \right] \left[\begin{array}{c} N \\ \\ \\ \end{array} \right]$$

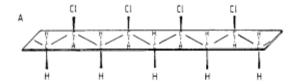
Figure 2. Examples of (a) rigid and (b) semi-rigid polymer molecules. A, poly(p-phenylene); B, poly(p-phenylene benzobisthiazole); C, poly(p-phenylene benzimidazole); D, poly(ether ether ketone); E, some thermotropic copolyesters.

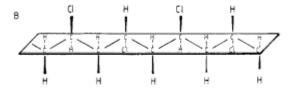


Rigid, semi-Rigid, Flexible Polymer Chains

On melting the entropy gain is small because the crystallizing units are chemically bound. The random coil state is seen classically as the highest entropy state. For rigid polymers the ground state may be a lower entropy state.

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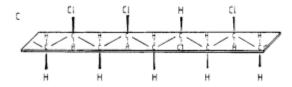


Figure 3. Major tacticity types in poly(vinyl chloride). A, isotactic; B, syndiotactic; C, atactic.

Tacticity

Tacticity governs the helical structure

Paul Phillips 1990 Review

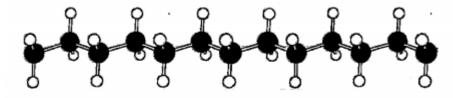
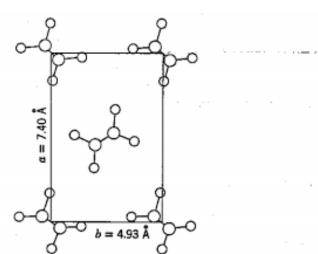


Fig. 2.1. The steric structure of PE. Rotations about the C-C bonds result in a change in the conformation



aguordi motiau 87.500-1 7.300 7.200

Fig. 3.7. Lattice constants thermal expansion for linear polyethylene (Davis et al.,24)

Figure 6-40 Arrangement of the molecules of polyethylene in the c-projection. (Courtesy of E. S. Clark.)

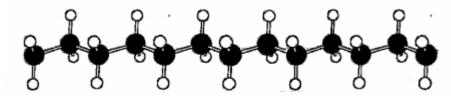
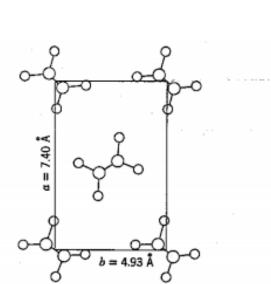


Fig. 2.1. The steric structure of PE. Rotations about the C-C bonds result in a change in the conformation



The effect of drawing on lattice constants (PE) depth of the polymers like polyethylene terephralate (PE) depth of the polymers like polyethylene terephralate (PE) depth of the polyethylene terephra

Fig. 3.18. a- and b-lattice constants of PE as a function of draw ratio (Glenz et al.(47);reproduced with permission by John Wiley & Sons, publishers.).

Figure 6-40 Arrangement of the molecules of polyethylene in the c-projection. (Courtesy of E. S. Clark.)

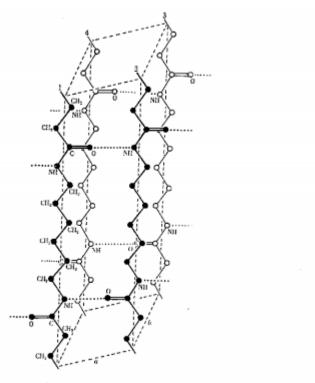


Figure 1-5 Arrangement of molecular chains in nylon 66, poly(hexamethylene adipamide). (Bunn and Garner[6].)

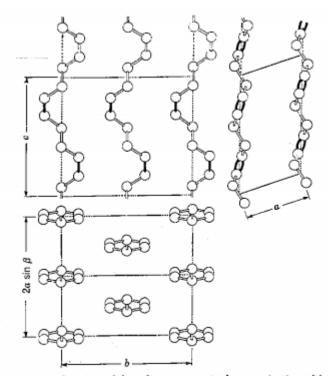


Figure 1-6 Structure of cis 1,4-polybutadiene as seen in three projections. Monoclinic unit cell with a=4.60, b=9.50, c=8.60 Å; $\beta=109^\circ$. (Natta and Corradini [7].)

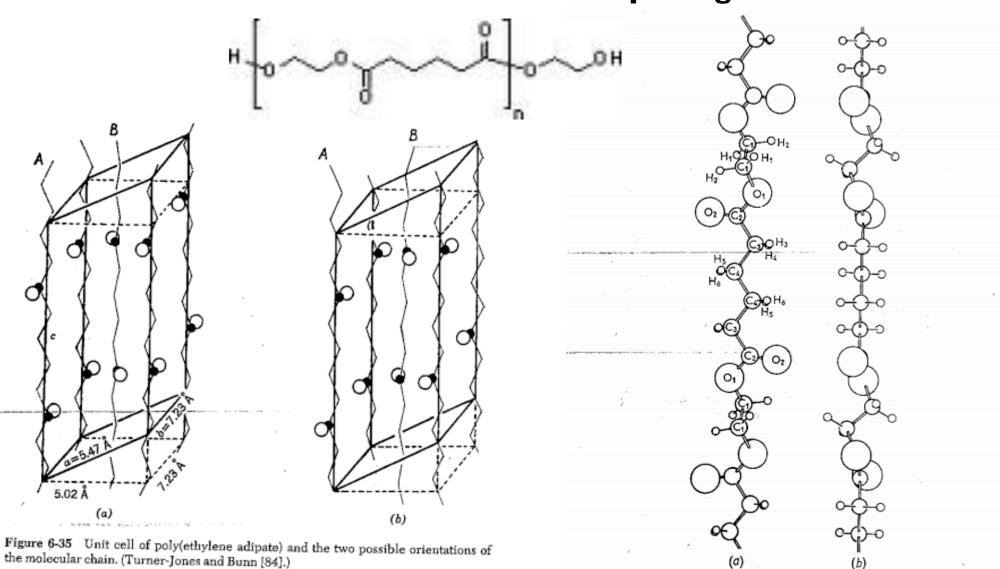


Figure 6-37 Chain configuration of poly(ethylene adipate) viewed (a) perpendicular and (b) parallel to the plane of the adipate chain. (Turner-Jones and Bunn [84].)

Figure 6-38 Arrangement of the molecules in the c-projection. (Turner-Jones and Bunn [84].)

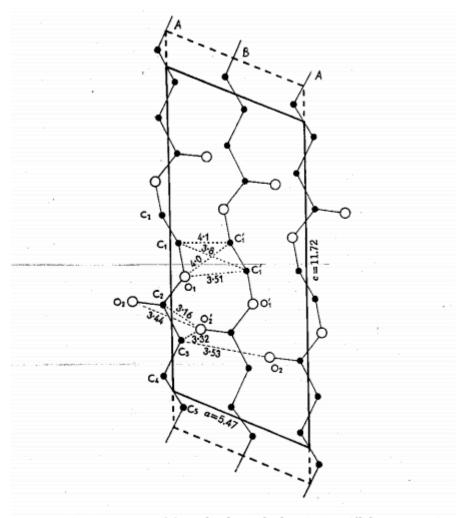
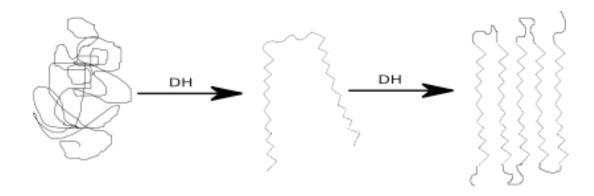
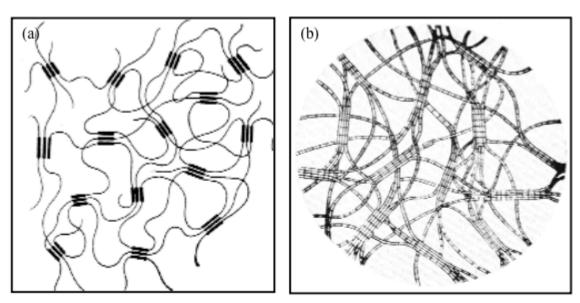


Figure 6-39 Arrangement of the molecules in the b-projection. All distances are in angstroms. (Turner-Jones and Bunn [84].)

Chain Folding and Crystallization





Fringed micelle model (a) Model of crystallization as might be visualized in a thermoreversible gel (Keller et al¹⁰.) (b) Hermann and Gerngross model¹⁵ for a semicrystalline polymer. Similar schematics illustrate the general molecular picture in fringed micellar crystallization.

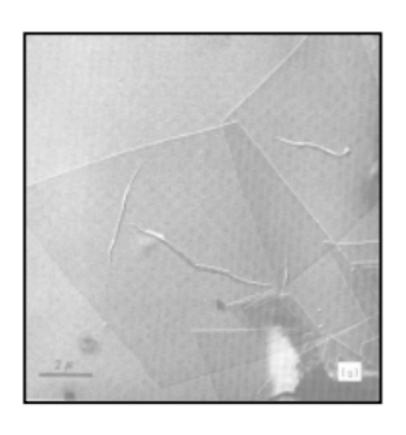
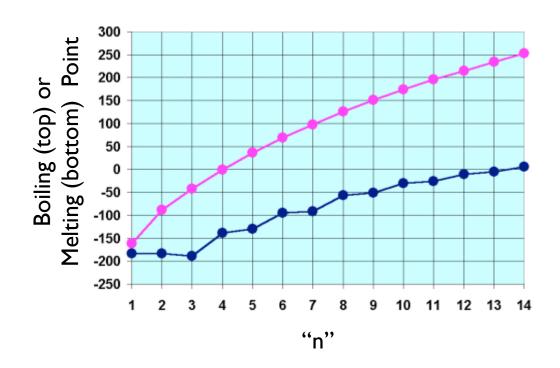


Figure 2.6 Single crystals of Polyethylene after evaporation of tetrachloroethylene solvent. Pleats form due to crystal collapse. Micrograph is taken from 'Polymer Single Crystals' by P.H. Geil¹⁹.



n-alkane boiling and melting points

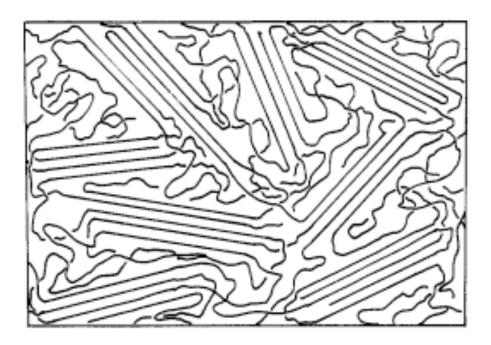
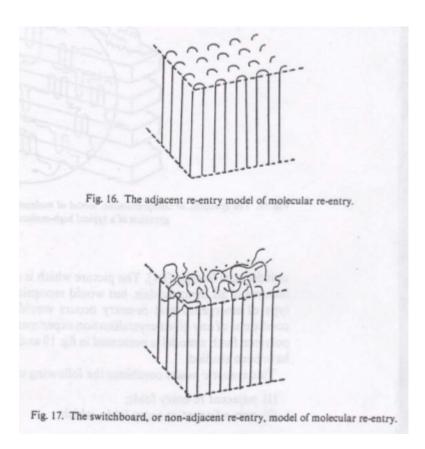
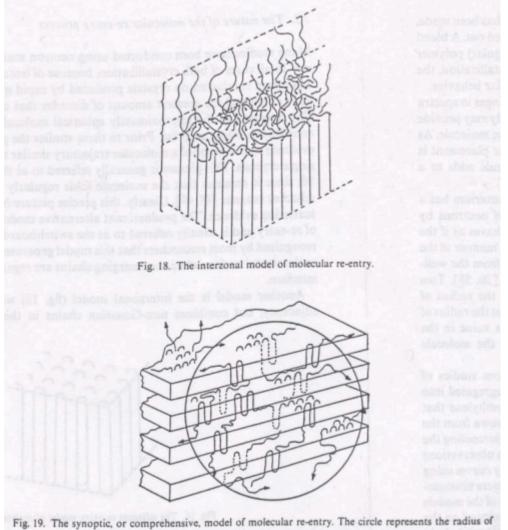
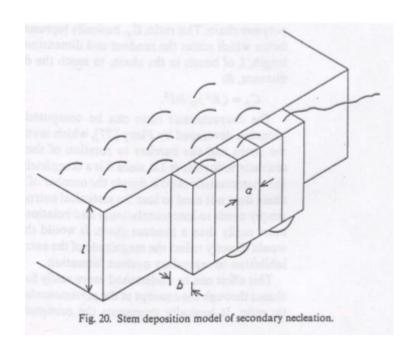


Figure 4. Schematic diagram of the fringed micelle model of polymer crystals.

"Crystals act like crosslinks in rubber" P. Phillips 1990







(1) Random reentry or "Switchboard" folded model.

This model was first proposed by Flory^{17,29,30,31} and consists of chains randomly folding back into the same lamella or even participating in adjoining lamellae. The upper and lower surfaces consist of loops of varying sizes and the amount of adjacent reentry is *small* and *not a necessity*^{17,29,30,31}. The upper and lower surfaces may consist of transitional regions that constitute a diffuse phase boundary – their density being intermediate between the crystal and purely amorphous regions.

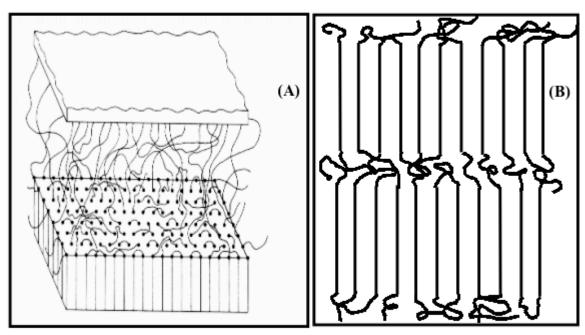
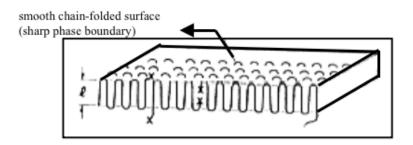


Figure 2.7 (A) Schematic of a Switchboard model, showing the surface of a lamella, interlamellar region and tie chains between the lamella. (From Mandelkern³⁰) (B) originally proposed model for melt crystallization in polymers¹⁷.

(2) Adjacent reentry chain-folded models (regular folding)

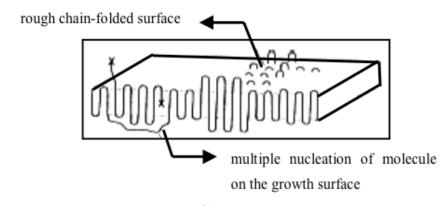
Smooth surface model^{32,33}:

This model is characterized by sharp phase boundary between the crystal and the amorphous phase. The mode of reentry of the chains is the adjacent neighbor with only a few exceptions due to multiple nucleation and chain-end defects. This is a very idealized visualization of the chain folding process.



(ii) Rough surface model³³:

The reentry of the chain is still in the nearest growth plane, though large variations in the fold length may exist on a local scale. Multiple nucleation and chain-end defects will further contribute to a rough surface. The overall phase boundary is no longer sharp, though local regions may still exhibit such character.



(3) "Erstarrungsmodell" (solidification model)^{34,35}

This model was put forward by Fischer & coworkers to explain the constancy of the radius of gyration r_g in the crystalline state (with respect to r_g in the amorphous state), as detected by small angle neutron scattering (SANS)³⁶. The model is similar in conception to the "fringed micellar" morphology and is visualized in terms of alignment of chains without a long-range diffusion process to give rise to a lamellar morphology. The chain sequences in proper conformations (indicated by thicker lines in the diagram) are incorporated into the crystal without significant reorganization of the chain conformation.

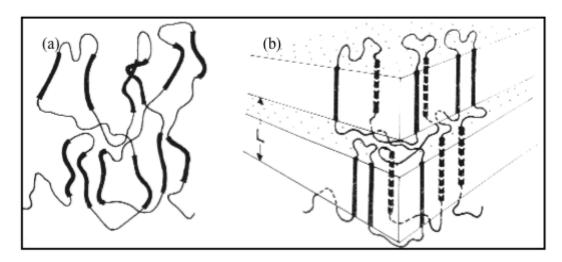
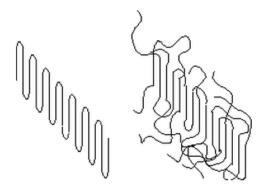


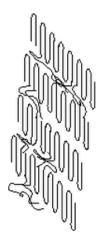
Figure 2.8 "Erstarrungsmodell" (a) chain conformation in the melt state (b) alignment of suitable conformations in to the crystal³⁶.

Nature of the Chain Fold Surface:

In addition to determination of T_°, the specific nature of the lamellar interface in terms of molecular conformation is of critical importance to the Hoffman analysis. There are several limiting examples, 1) **Regular Adjacent Reentry**, 2) **Switchboard Model** (Non-Adjacent Reentry), 3) **Irregular Adjacent Reentry** (Thickness of interfacial layer is proportional to the temperature).



The **synoptic or comprehensive model** involves interconnection between neighboring lamellae through a combination of adjacent and Switchboard models.



The **interzonal model** involves non-adjacent reentry but considers a region at the interface where the chains are not randomly arranged, effectively creating a three phase system, crystalline, amorphous and interzonal.

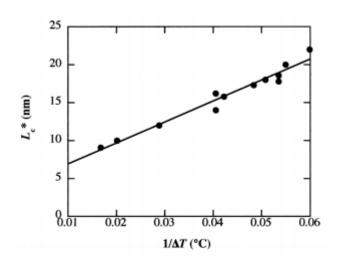


Fig. 6.11. Initial crystal thickness as a function of the reciprocal degree of supercooling (ΔT). Drawn after data of Barham at al. (63).

Solution Crystallization

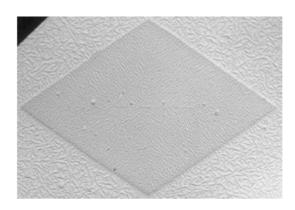


Fig. 6.4. Transmission electron micrograph of a replicate of a single crystal of polyethylene decorated with polyethylene vapour. With permission from Wiley, New York (30).

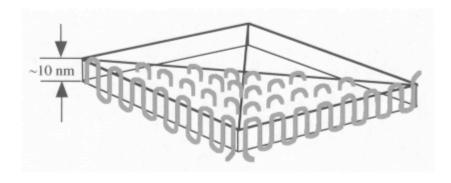
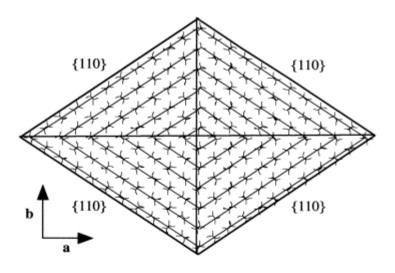


Fig. 6.5. Schematic drawing of single crystal with regular chain folding



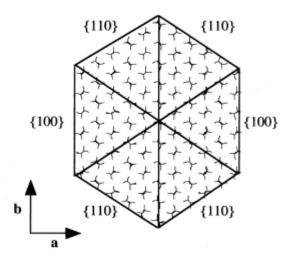


Fig. 6.6. Sectorization of polyethylene single crystals. The {110} sectors whereas the lower also has {100} sectors Kluwer, Doordrecht, the Netherlands (120).

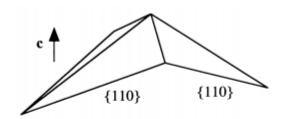
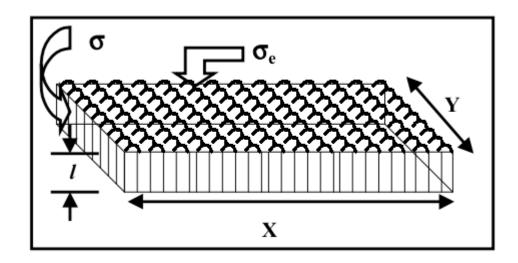


Fig. 6.7. Schematic drawing of tent-shaped polyethylene single crystals with only {110} sectors.



http://scholar.lib.vt.edu/theses/available/etd-051799-162256/unrestricted/polyimide2.pdf

$$V\Delta G_{Crystallization} = V\Delta H_{Crystallization} - VT\Delta S_{Crystallization} - S\sigma$$

For an infinite crystal V/S is very large and we can ignore the surface At the equilibrium melting point $\Delta G = 0$ so:

$$T_{\infty} = \frac{\Delta H_{Crystallization}}{\Delta S_{Crystallization}}$$

substituting in the first equation,

$$V\Delta G_{Crystallization} = V\Delta H_{Crystallization} \left(\frac{T_{\infty} - T}{T_{\infty}} \right) - S\sigma$$

$$V\Delta G_{Crystallization} = V\Delta H_{Crystallization} \left(\frac{T_{\infty} - T}{T_{\infty}} \right) - S\sigma$$

For a lamellar crystal S is $2R^2$ and V is tR^2

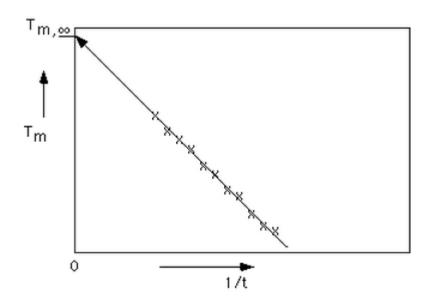
At a pseudo-equilibrium point where the crystal has a finite thickness of t and a depressed melting point, T

$$0 = t\Delta H_{Crystallization} \left(\frac{T_{\infty} - T}{T_{\infty}} \right) - 2\sigma$$

$$t = \frac{2\sigma T_{\infty}}{\Delta H_{Crystallization} \Delta T}$$

$$t = \frac{2\sigma T_{_{\infty}}}{\Delta H_{\mathit{Crystallization}}\Delta T} \qquad \text{or} \qquad$$

or
$$T_{m} = T_{\infty} - \frac{2\sigma T_{\infty}}{t\Delta H_{Crystallization}}$$

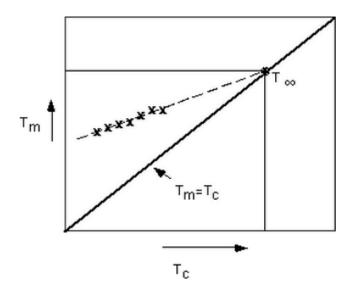


Hoffman-Weeks Plot

Hoffman-Weeks theory is based on the difference between melting, T_m , and crystallization , T_c , temperatures. In the Hoffman-Weeks approach, it is considered that any crystallite formed at temperatures less than T_c has some imperfections "locked-in". Generally, the melting temperature is higher than the crystallization temperature, $T_m > T_c$. Hoffman and Weeks defined a stabilization parameter, f', which is zero for crystallites with no imperfections and has a value of 1 for an unstable crystal with all imperfections. For the most stable crystallite $T_m = T_c$. For the completely unstable crystallite, $T_m = T_c$. For a typical crystallite, f'=1/2. Using a simple weighting law, Hoffman and Weeks wrote a linear expression relating T_m , T_c and T_c ,

$$T_m = T \circ (1 - f') + f' T_c$$

This linear law implies plots of T_m versus T_c to obtain T_c without requiring a measurement of the crystallite thickness. At T_c , $T_m = T_c = T_c$ according to this approach. The Hoffman-Weeks approach and the more direct approach yield very close values for T_c .



Hoffman-Weeks plot for To.

http://www.eng.uc.edu/~gbeaucag/Classes/MorphologyofComplexMaterials/Chapter2html/Chapter2.html

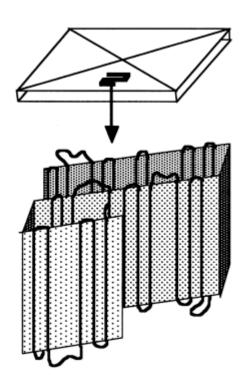


Fig. 6.13. The superfolding model according to Sadler and Keller (84) showing the conformation of a single molecule in a solution-grown single crystal.

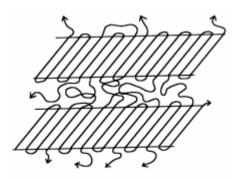


Fig. 6.14. Schematic description how overcrowding of the amorphous phase is avoided by adjacent, regular folding.

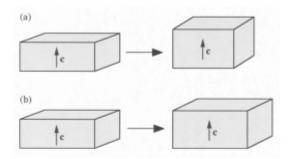


Fig. 6.9. Schematic description of crystal thickening options: (a) Crystal thickening at constant crystal volume. (b) Crystal thickening accompanied by an increase in crystal volume.

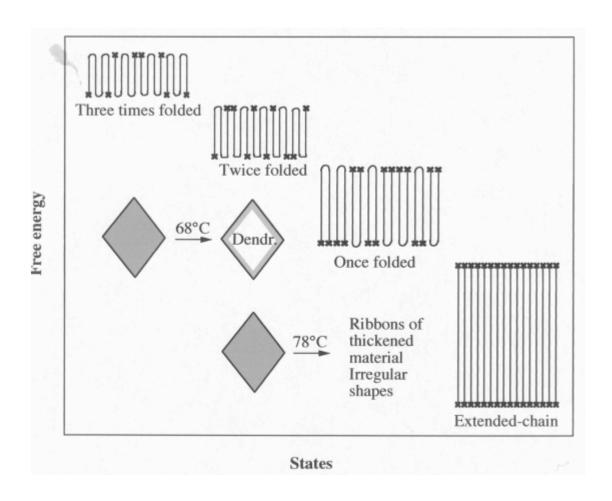


Fig. 6.10. Schematic representation of crystal thickening of $C_{294}H_{590}$. Drawn after findings of Hobbs et al. (51).

P J Phillips

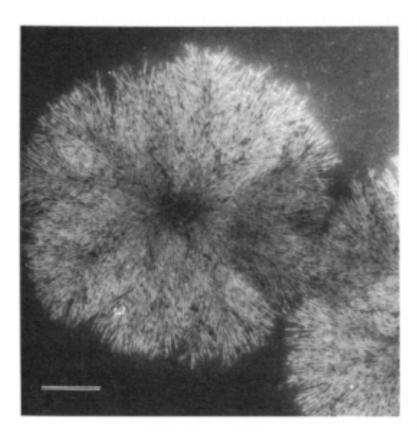


Figure 5. A spherulite growing in a film of cis-polyisoprene. The crystal growth has been terminated prior to completion, through reaction of the film with osmium tetraoxide vapour, thereby permitting resolution of the individual lamellar crystals. (Scale bar: $0.5 \mu m$.) First published by Phillips (1983).

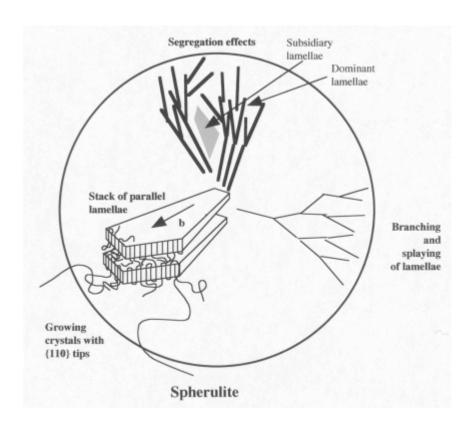
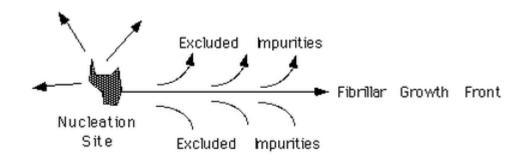


Fig. 6.8. Schematic drawing of the morphological hierarchy of a polyethylene spherulite.



Diffusion of impurities versus growth rate of crystallization front

J = -D dc/dx for flux of impurities G = linear growth rate of crystal

D/G = δ , the Keith-Padden δ -parameter

This determines the coarseness of the spherulite (lateral size)

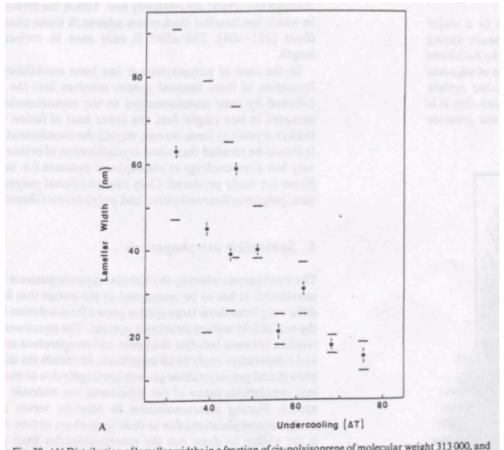


Fig. 28. (A) Distribution of lamellar widths in a fraction of cis-polyisoprene of molecular weight 313 000, and (B) dependence of lamellar width on supercooling at (a) -20° C and (b) -0.6° C (molecular weight 543 000) [108].

The original L-H theory and its various modifications account for a broad range of behavior observed for crystallization of linear flexible macromolecules. These are³⁹:

- it accounts for the variation of initial lamellar thickness (1*) vs. supercooling (ΔT_c)
- parameters can be found that fit the variation of crystal growth rate 'G' vs. ΔT_c
- provides an explanation for break in temperature dependence of G
- explains the origin of σ and σ_e
- the generation and effect of adjacent events (tight folding) and non-adjacent events (e.g. tie chains, loose folds, cilia)
- Variation of (a) the crystal growth rate 'G' and (b) quantified chain folding (i.e. degree of tight chain folding), with the change in molecular weight
- Recent versions have also incorporated the 'reptation' concept into the theory

The various facets of polymer crystallization still not addressed completely by the theory are:

- Explanation for primary nucleation and hence bulk crystallization kinetics
- Development of lamellae from a primary nucleus
- Lamellar branching giving rise to spherulites (other factors like screw dislocations provide some explanation)
- Banding in spherulites due to lamellar twisting
- Quantified estimation of the degree of crystallinity

Add a first stem, then subsequent stems.

First stem and subsequent stems add

 $2b_0\sigma I$ and $b_0\sigma I$

To the free energy (cost to make a surface)

First stem:

Also the change in free energy when going from subcooled melt to the activated stem is:

$$\Delta G_{c}^{*} = 2b_{0}\sigma l + \psi [a_{0}b_{0}l] \Delta G_{c}$$
 {2.18}

where ΔG_c is the free energy of crystallization/unit volume and ' ψ ' is the apportionment factor, i.e. the fraction of free energy available due to the crystallographic attachment at a finite and few number of sites. The possible values of ' ψ ' can thus be between 0 and 1.

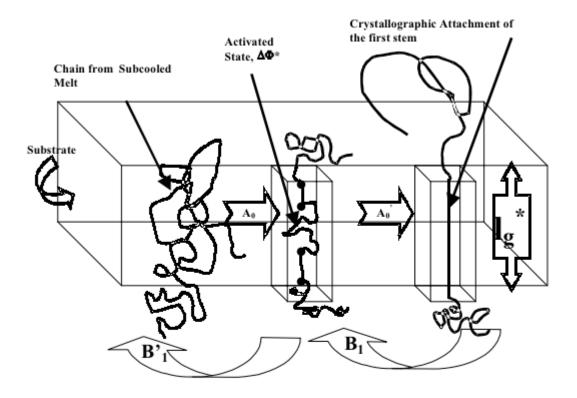


Figure 2.9 Formation of the physically aligned activated complex and its conversion to first crystallographically attached stem. The first step A_0 is the slowest and rate determining step while the step A_0 is fast³⁹.

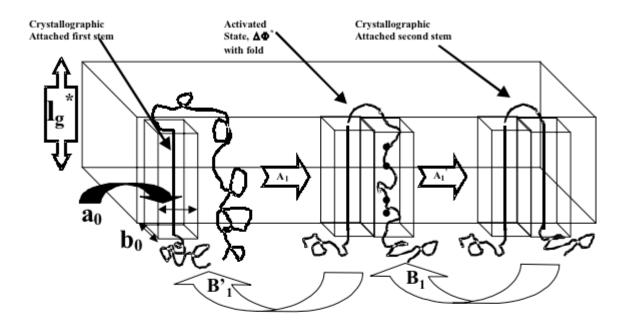


Figure 2.10 Formation of the physically aligned activated complex and its conversion to second crystallographically attached stem. The first step A_0 is the slowest and rate determining step while the step A_0 is fast. The activated state includes a tight fold + the aligned part of the chain³⁹.

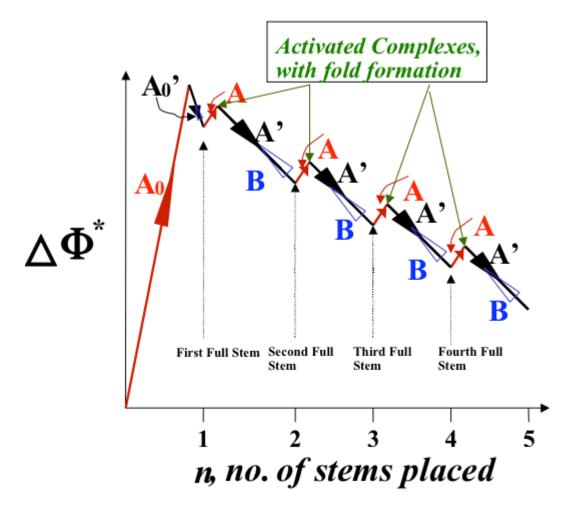


Figure 2.11 Barrier system for the surface nucleation showing both the slow, fast and backward steps possible. A, A₀, B₁ and B are the rate determining slow steps while A₀' and A' are the fast steps³².

The above-discussed concepts lead us to calculations involving the lateral substrate completion rate, S_T as given by equation (2.35) below. The steady state flux S_T is

$$S_T = \frac{1}{l_u} \int_{2\sigma_e/\Delta G}^{\infty} S(l) dl$$
 {2.35}

where lu is the monomer length.

This allows us to calculate the rate of stem deposition 'i', i.e. the surface <u>nucleation</u> rate in terms of <u>stems s⁻¹ cm⁻¹</u> by the simple relation³⁹

$$i = S_T / L = S_T / n_i a_0$$
 {2.36}

where n₁ is the number of stems of width a₀ which make up the substrate of length L. The

second important parameter leading up to the crystal growth rate 'G' is given by the substrate completion rate 'g'. The substrate completion rate is given by

$$g = a_0 \text{ (A-B)}$$
 {2.37}

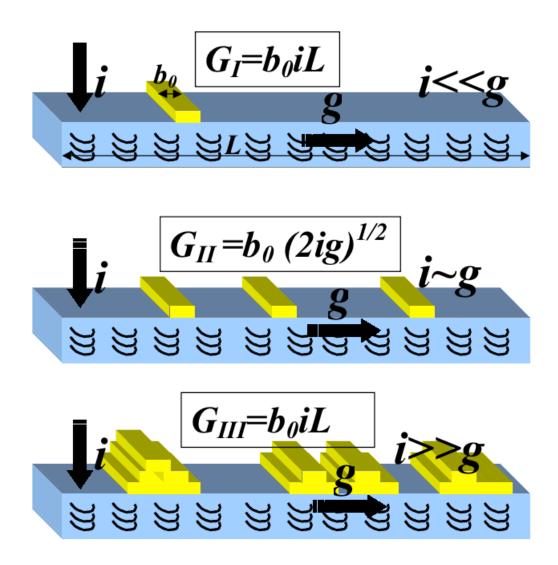


Figure 2.13 Scheme illustrating the rates of stem deposition during three different regimes of crystallization. 'i' represents the rate of stem nucleation whereas 'g' represents the rate of substrate completion.

$$i = \frac{N_0 \beta}{n_1 a_0 l_u} \left[\frac{kT}{2b_0 \sigma} - \frac{kT}{2b_0 \sigma + \Delta G_f} \right] \exp \left[\frac{-4b_0 \sigma_e \sigma}{\Delta G_f kT} \right]$$

 $\{2.38\}$

$$g = a_0 \beta \left[1 - \exp\left(\frac{a_0 b_0 \delta l \Delta G_f}{kT}\right) \right] \exp\left[\frac{-2 a_0 b_0 \sigma_e}{kT}\right]$$

$$\beta = J \exp\left[\frac{-U^*}{R(T - T_{\infty})}\right] \qquad J = \frac{\kappa}{n} \left(\frac{kT}{h}\right)$$

$$G_I = G_{0I} \exp \left(-U^*/R \left(T-T_{\infty}\right)\right) \exp \left(-K_{gI}/T\Delta T_c\right)$$

 $G_{II} = G_{0II} \exp \left(-U^*/R \left(T-T_{\infty}\right)\right) \exp \left(-K_{gII}/T\Delta T_c\right)$
 $G_{III} = G_{0III} \exp \left(-U^*/R \left(T-T_{\infty}\right)\right) \exp \left(-K_{gIII}/T\Delta T_c\right)$

$$G_{0i} = \frac{N_0 b_0 J}{l_u} \left[\frac{kT}{b_0 \sigma} - \frac{kT}{2b_0 \sigma + a_0 b_0 \Delta G_f} \right]$$

$$K_{gI} = K_{gII} = 2K_{gIII} = \frac{4\sigma\sigma_e T_m}{k\Delta H_f}$$

The value of the nucleation constants K_{gl} , K_{gIII} , K_{gII} and G_{0i} can thus be determined by plotting the spherulitic growth rate data in the form LnG+ U*/R (T-T_∞) vs. 1/ T Δ T_c. These types of plots are usually referred to as L-H plots and Figure 2.13 illustrates the typical plots for polymers showing these transitions. The first exponential term in equation (2.40), exp (-U*/R (T-T_∞)) accounts for the chain transport effects to the interface while the second term exp (-K_{gi}/T Δ T_c), accounts for the secondary nucleation effects. The widely utilized values for U* and T_∞ are 1500 cal/mol and T_g-30K for a large number of polymers^{39,43}. L-H plots have also been widely utilized to obtain values for σ _c if the values of T_m°, Δ H_f and b₀ are known^{40,58,59,60}, or otherwise sometimes to obtain T_m° ^{54,61,62}.

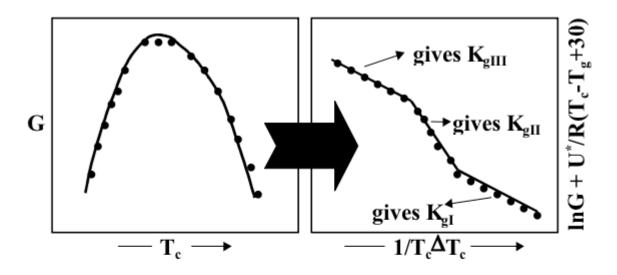


Figure 2.14 A schematic illustrating the conversion of growth rate data to a L-H plot showing the three regime transitions³⁸. The values of regime constants are calculated by the slope in various regimes and are used to give the product of surface energy terms $\sigma\sigma_e$. All three regimes or even a single regime transition may not be experimentally observed for many polymers.

Primary Nucleation

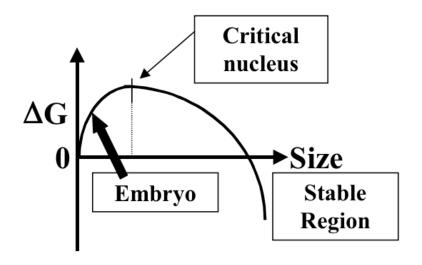


Figure 2.15 Schematic illustrating the variation of free energy with nucleus size.

The initial free energy barrier needs to be crossed for the nucleus to become stable⁵.

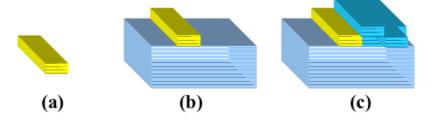


Figure 2.16 Types of Crystal Nuclei (a) Primary nucleus (b) Secondary Nucleus (c) Tertiary Nucleus

Primary Nucleation

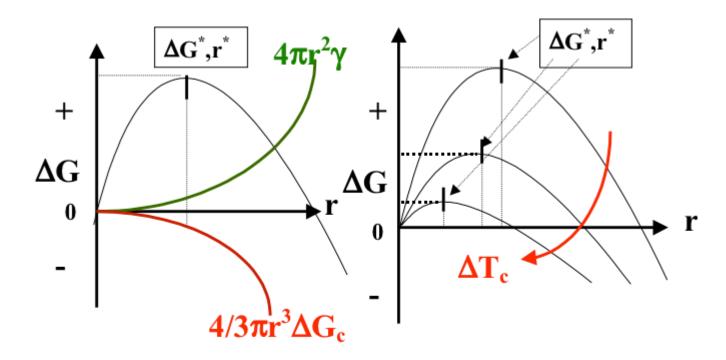


Figure 2.17 Variation of total free energy with size depends upon two opposing factors, the gain being due to increased surface area while the loss due to free energy of crystallization. Also, the critical size for stable nuclei formation as well as the critical free energy barrier decrease with increasing undercooling^{2,38}.

Spherulites

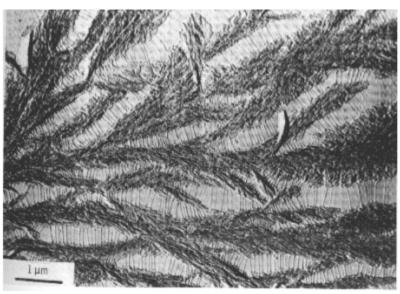
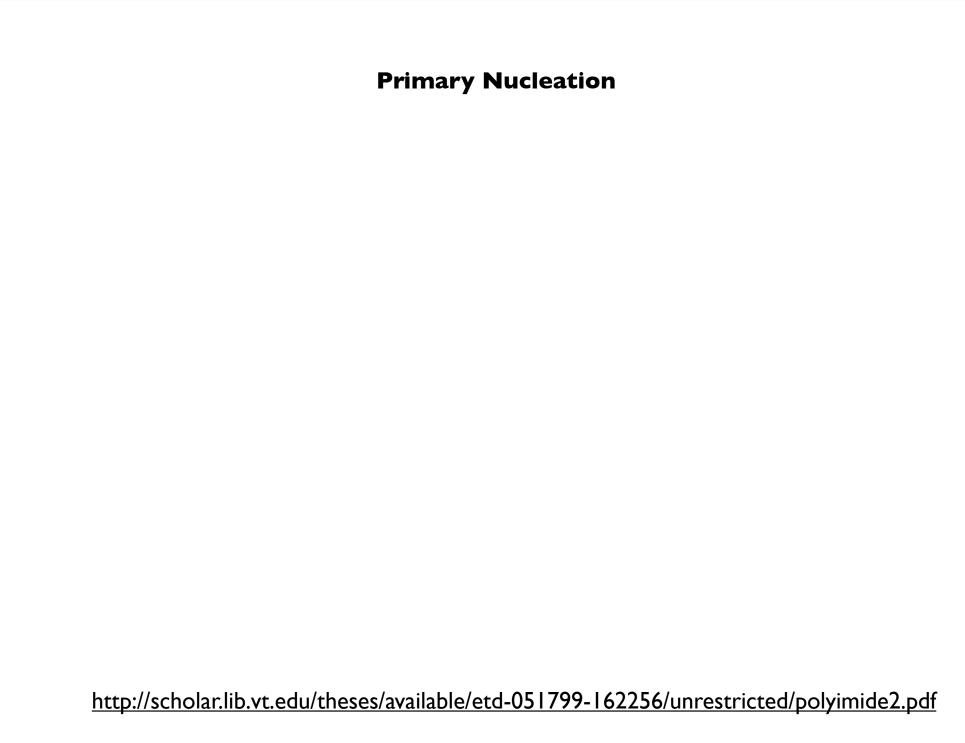
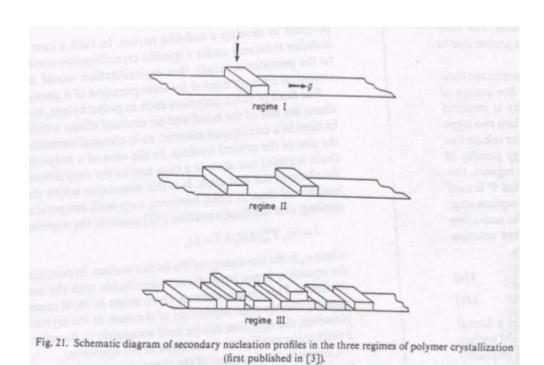
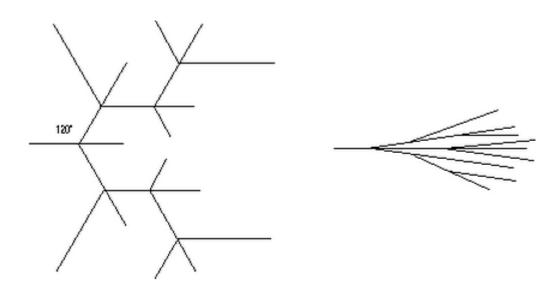


Figure 2.18 Tie chains in polyethylene spherulites crystallized in presence of n-parafin, C₃₂H₆₆, and then extracted with xylene at room temperature.

(Keith and Padden⁷⁰ et al.)







ttp://www.eng.uc.edu/~gbeaucag/Classes/MorphologyofComplexMaterials/Chapter2html/Chapter2.html

Non Crystallographic Branching

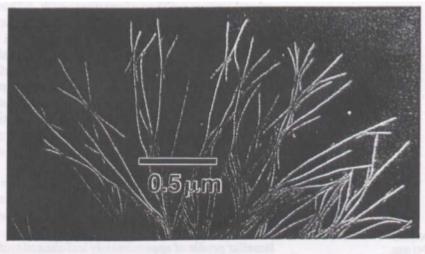


Fig. 29. Non-crystallographic branching in cis-polyisoprene crystallized at -15°C.

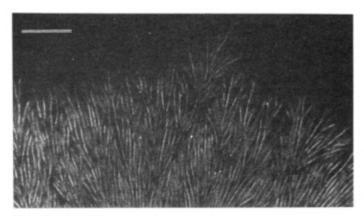


Figure 21. Leading lamellae protruding ahead of the growth front in a beta-spherulite of cis-polyisoprene. Due to extensive crystallographic branching, the protuberances are smaller than in spherulites of alpha-crystals of the same polymer. Scale bar $0.2 \, \mu m$. (Sorenson and Phillips, unpublished data).



Figure 19. Transmission electron micrograph of a shadowed carbon replica of permanganically etched linear polyethylene showing ridged crystals. The specimen ($M_w = 37\,000$, $M_n = 29\,000$) had been crystallised at 130 °C for 7.5 days followed by 30 min at 125 °C prior to a cold-water quench. (Bassett and Hodge, unpublished work. Photograph supplied by D C Bassett.)

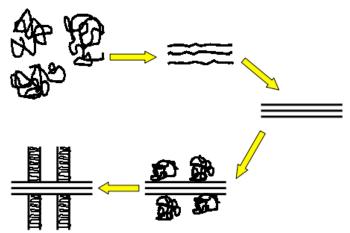


Figure 2.3 Schematic representation of orientation induced crystallization.

The first three drawings illustrate the orientation and crystallization of random coils while the last two drawings show the growth of folded chain kebabs around the central shish¹⁰.

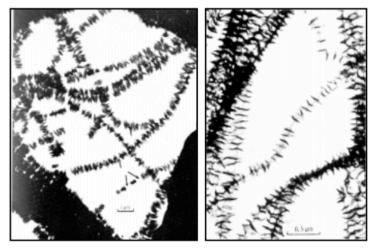


Figure 2.4 (a) Shish kebab morphology of polyethylene from solution (from Pennings, 1967³. (b) Shish kebabs of cellulose formed by recrystallizing cellulose II onto microfibrils of high molecular weight³.

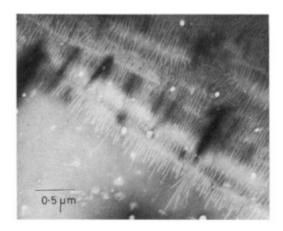
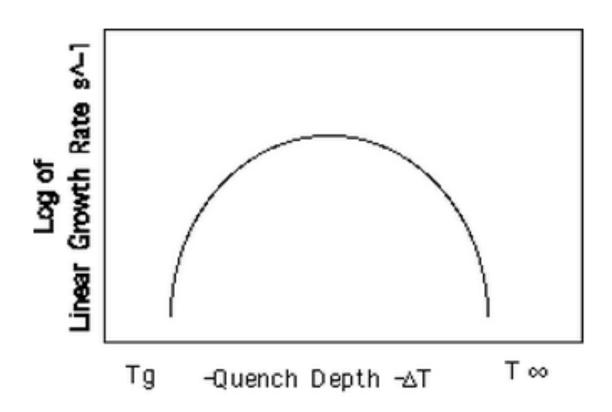


Figure 20. Row-nucleated growth (shish-kebabs) in cis-polyisoprene (Edwards and Phillips, unpublished data).

Spherulitic Growth Rate



Poisson⁸¹ in 1837, which states that for raindrops falling randomly, the probability of a point being passed over by exactly F wavefronts is given by

$$P(F) = \frac{e^{-\overline{F}}\overline{F}^F}{F!}$$
 {2.48}

where \overline{F} is the average number of such wavefronts passing through a point. Thus considering these wavefronts as spherulites in bulk crystallization, the probability of any point not being run over by a spherulite is given by value of P(F) at F=0. Thus

$$P(0) = e^{-\overline{F}} {2.49}$$

P(0) also represents the points which are still amorphous and not been run over by the spherulites and thus is equal to amorphous fraction 1- θ , where θ is the amount of fraction crystallized.

$$1 - \theta = P(0) = e^{-\overline{F}}$$

$$\Rightarrow \ln \frac{1}{1 - \theta} = \overline{F}$$

$$\Rightarrow \theta = 1 - \exp(-\overline{F})$$
(2.50)

Now the problem reduces to obtaining the form of the function \overline{F} for different types of geometries that may be involved. The time dependency of the crystalline fraction in the above analysis enters due to time dependency of the function \overline{F} , the average number of

wavefronts passing in time 't'. For some particular cases, this function can be calculated to give the following relations^{5,82}:

(a) 2-dimensional case of growing discs starting at the same time

$$\overline{F} = \pi G^2 N t^2 \tag{2.51}$$

where G is the growth rate of growing discs, N is the average number of such discs/area and t is the elapsed time.

(b) 2-dimensional case of growing discs forming at a rate \dot{N}

$$\overline{F} = \frac{\pi}{3} G^2 \dot{N} t^3 \tag{2.52}$$

(c) 3-dimensional case of growing spheres starting at the same time

$$\overline{F} = \frac{4}{3}\pi G^3 N t^3$$
 (2.53)

(d) 3-dimensional case of growing spheres forming at a rate \dot{N}

$$\overline{F} = \frac{\pi}{3} G^3 \dot{N} t^4$$
 {2.54}

In general then, the form of the equation is of the type

$$\theta = 1 - \exp(-Kt^n) \tag{2.55}$$

which is the famous Avrami equation and the 'K' & 'n' are the two Avrami parameters

Table 2.2. Avrami exponents for various types of crystal growth geometry's 82.

Avrami Exponent	Crystal Geometry	Nucleation Type	Rate Determination
0.5	Rod	Athermal	Diffusion
1	Rod	Athermal	Nucleation
1.5	Rod	Thermal	Diffusion
2	Rod	Thermal	Nucleation
1	Disc	Athermal	Diffusion
2	Disc	Athermal	Nucleation
2	Disc	Thermal	Diffusion
3	Disc	Thermal	Nucleation
1.5	Sphere	Athermal	Diffusion
2.5	Sphere	Thermal	Diffusion
3	Sphere	Athermal	Nucleation
4	Sphere	Thermal	Nucleation

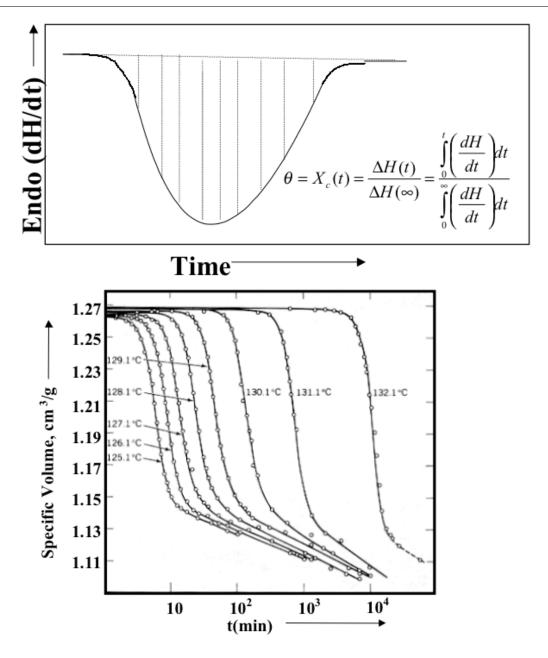


Figure 2.19 Utilization of isothermal crystallization data by either DSC or by volume³⁰ measurements can give the degree of transformation, which can subsequently be utilized for Avrami analysis.

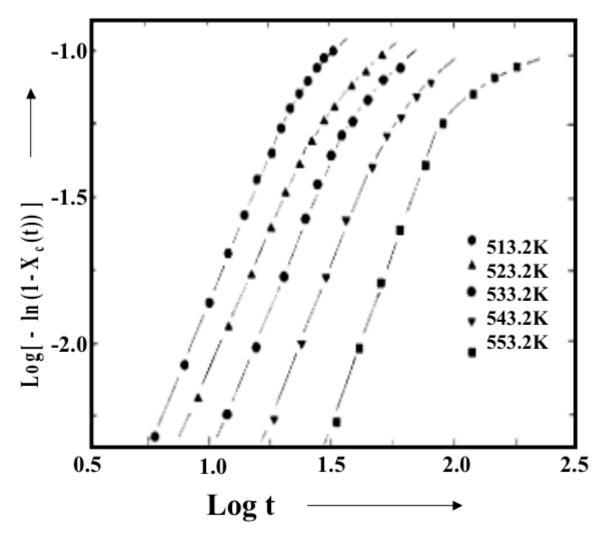


Figure 2.20 The characteristic Avrami plots obtained by isothermal crystallization experiments for a polyimide⁸⁴. The initial slope of the curves gives the Avrami constant 'n', which is related to the crystal shape and nucleation type.

- (a) The Avrami equation rigorously applies only to problems where the volume does not change. This is never the case with crystallization in polymers.
- (b) It assumes constancy in the shape of growing disc/rod/sphere
- (c) Constant radial growth is assumed (G~t-1/2 has also been considered)
- (d) The analysis does not account for the presence of an induction time
- (e) The nucleation mode is assumed to be unique i.e. thermal or athermal but not both
- (f) Complete crystallinity of the sample
- (g) Random distribution of nuclei
- (h) Constant value of radial density in the growing structures which is assumed in the derivation does not usually occur experimentally
- (i) Holds well for primary crystallization only
- (j) Does not account for absence of overlap between growing crystallization fronts. It is thus not surprising that non-integer values of n are often obtained. As shown in Table 2, it is not difficult to assign the experimentally obtained value of n by selecting an appropriate geometry. This kind of attribution of the exponent 'n', without independent microscopical evidence is one of the major pitfalls of most studies in the literature utilizing this analysis. Independent microscopical evidence is critical before assignment of 'n' to a particular geometry can be justified.