021101 Quiz 4 Mechanics of Materials

1) -For simple shear give the relationships between the three Cartesian coordinates in the deformed and undeformed states.
-Calculate the Finger tensor using these relationships.
-Use this Finger strain tensor and the Finger equation to calculate the stress strain relationship for the shear stress and the first normal stress difference.

2) The following plot is based on the Mooney-Rivlin equation for a tensile deformation.

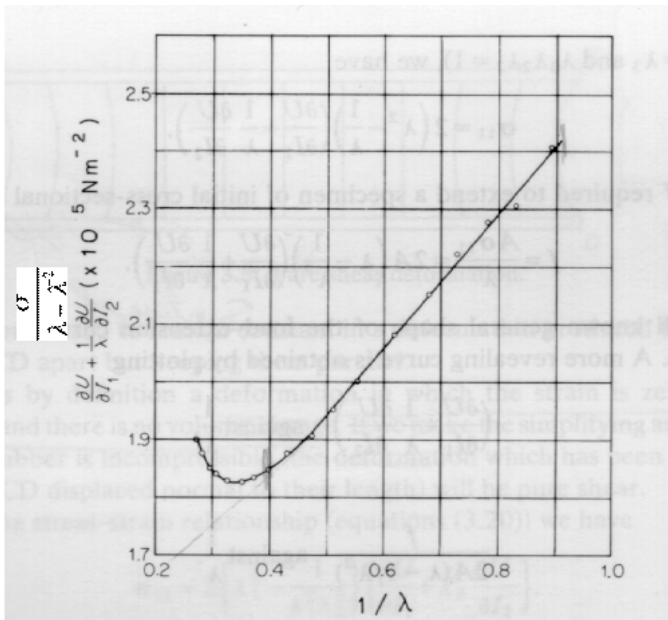

Figure 3.7. Plot of $[\partial U/\partial I_1 + (1/\lambda)\,\partial U/\partial I_2]$ against $1/\lambda$ from experiment in simple extension. [Redrawn with permission from Rivlin and Saunders, *Phil. Trans. Roy. Soc. A*, **243**, 251 (1951).]

-Is this plot obtained from the ideal rubber equation,

$$\sigma_{11} = \rho k T \left(\lambda^2 - \frac{1}{\lambda} \right)$$

-If yes explain how, if no explain why not.

3) -Explain the term "Memory Function" when referring to G(t) in the following expression,

$$\sigma(t) = -\int_{t^* = -\infty}^{t} G(t - t^*) \frac{d\mathbf{B}(t, t^*)}{dt^*} dt^*$$

-Give this memory function for a Hookean elastic, Newtonian fluid and a viscoelastic.

4) -Plot $\eta(\dot\gamma)$, $\Psi_1(\dot\gamma)$ and $\gamma_e(\dot\gamma)$ on a log-log scale.
-Show where power-law fluid behavior is observed and
-where the values η_0, and Ψ_{10} are determined.

Answers: 021101 Quiz 4 Mechanics of Materials

1) For simple shear deformation, we can write in Cartesian coordinates the relationship between the deformed and undeformed (primed) states,

$$x' = x - \gamma z$$
$$y' = y$$
$$z' = z$$

B_{ij} is given by $B_{ij} = \Sigma e_{i\mu} e_{j\mu} = \Sigma\ \delta r'_i/\delta r_\mu\ \delta r'_j/\delta r_\mu$. For the xx component $B_{11} = 1x1 + 0x0 + \gamma$ x γ. The xz component is B_{13} = 1 x 0 + 0 x 0 + γ x 1.

$$B_{ij} = \begin{pmatrix} 1+\gamma^2 & 0 & \gamma \\ 0 & 1 & 0 \\ \gamma & 0 & 1 \end{pmatrix}$$

This is used in the Finger Equation, $\underline{\underline{\sigma}} = G \cdot \underline{\underline{B}} - P\underline{\underline{1}}$, to yield,

$$\sigma_{xy} = (n_c/V)\ kT\ \gamma$$

$$\sigma_{xx} = (n_c/V)\ kT\ (1+\gamma^2) - P$$

$$\sigma_{yy} = \sigma_{zz} = (n_c/V)\ kT - P$$

$\psi_1 = (\sigma_{xx} - \sigma_{zz}) = G\ (\gamma')^2$. The shear modulus is given by,

$$G = \lim_{\gamma \to 0} \frac{\sigma_{xy}}{\gamma} = \nu_c kT$$

where ν_c is the number density of network chains. Shear deformation of an elastomer leads to a simple linear Hookean expression for the modulus in contrast to the tensile expression given above.

2) The plot can not be obtained from the ideal rubber equation. As indicated on the axis (plot is from I. M. Ward, Mechanical Properties of Solid Polymers) the y-axis and the Mooney-Rivlin equation use both the first and second invariants of the Finger tensor in calculating the stress strain relationship. The ideal rubber equation uses only the first invariant to calculate the equation for tension. The use of the second invariant in the Mooney-Rivlin equation allows for non-ideal, enthalpic effects to be measured.

3) G(t) is the time dependent modulus in a stress relaxation measurement. In the stress relaxation measurement a fixed strain, γ_0, is applied to a sample and the decay of stress is observed and used to calculate $G(t) = \sigma(t)/\ \gamma_0$. For a Hookean solid the response is a step

function (like a rectangle), for a Newtonian liquid the response is a delta function (a spike to η), and for a viscoelastic or relaxing system the response is a spike followed by an exponential decay. G(t) reflects the stress memory of strain applied to a material at t=0 and observed at time t. In this way it is a memory function. In application we consider that all incremental strains applied to the material over time, t*, can be summed to calculate the stress at the present time, t, if weighted by the memory function, G(t-t*). (t-t*) ensures that we consider the time difference between application of strain and observation of stress. If this is written as an integral we integrate over the derivative of the Finger strain tensor at time t* in the context of the present time t. If the three memory functions, Hookean, Newtonian and viscoelastic, are used we find Hooke's law, Newton's law and a definition of viscosity as the integral of the memory function for the three cases. The equation also gives the dependence of the first normal stress on the square of the velocity gradiant (rate of strain) as anticipated by our considerations of static stress strain for hyper-elastic materials.

4)

328 Chapter 7 Non-Linear Mechanical Behavior

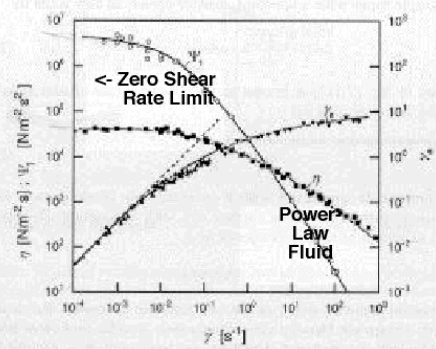

Fig. 7.15. PE under steady state shear flow at 150 °C: Strain rate dependencies of the viscosity η, the primary normal stress coefficient Ψ_1 and the recoverable shear strain γ_s. The *dotted line* represents Eq. (7.122). Results obtained by Laun [76]