Quiz 1 Mechanics of Materials
020930

1) a) Demonstrate that $\sigma_{12} = \sigma_{21}$ by making a sketch of a Cartesian coordinate system with two shear stresses and a torsional element located midway between the two shear stresses.

b) What assumptions are necessary for this equality to be true?

2) a) Show that $\frac{1}{2}\left(\sigma_{ik}\sigma_{ki} - \sigma_{ii}\sigma_{kk}\right) = \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix}$ by expanding the determinates and expanding the summations.

b) What quantity do these expressions describe.

c) What is an invariant?

3) a) How many independent components are there in the stress tensor?

b) How many independent components are there in the following tensors:

Displacement tensor, e_{ij}
strain tensor, ε_{ij}
rotational tensor, ω_{ij}

c) If the stress is related to strain through a tensoral modulus, $\sigma_{ij} = C_{ijkl}\varepsilon_{ij}$, how many components would this generic modulus have?

Answers: Quiz 1 Mechanics of Materials

1) a) This can be demonstrated for one pair of symmetric shear stresses, for instance σ_{xy} and σ_{yx}, Figure 2. The dashed line in figure 2 is a torsional bar which must have not rotational torque is the system is not subjected to rotational motion. Two stresses, σ_{xy} and σ_{yx}, are applied to the system. The torsional element is subjected to two rotational forces shown. These two forces must balance under these conditions leaving $F_y = F_x$ and $\sigma_{xy} = \sigma_{yx}$. (For each off diagonal torsional element the solid body has two symmetric torsional elements which when summed lead to the same result.)

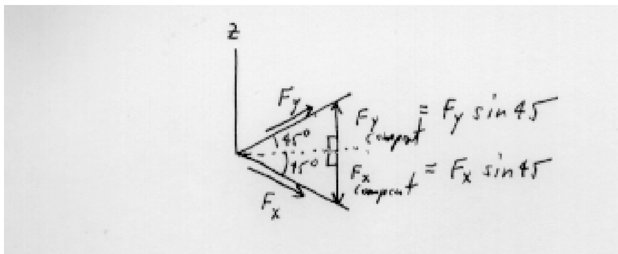

Figure 2. Demonstration That $\sigma_{xy} = \sigma_{yx}$
Dashed Line is a torsional Element

b) Assumptions: No rotational torque on the material.

2) a) $\frac{1}{2}\left(\sigma_{ik}\sigma_{ki} - \sigma_{ii}\sigma_{kk}\right) = \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix}$

For the Einstein notation expression:

$$\frac{1}{2}\begin{bmatrix} \left(\sigma_{11}\sigma_{11} - \sigma_{11}\sigma_{11} + \sigma_{12}\sigma_{21} - \sigma_{11}\sigma_{22} + \sigma_{13}\sigma_{31} - \sigma_{11}\sigma_{33}\right) \\ +\left(\sigma_{21}\sigma_{12} - \sigma_{22}\sigma_{11} + \sigma_{22}\sigma_{22} - \sigma_{22}\sigma_{22} + \sigma_{23}\sigma_{32} - \sigma_{22}\sigma_{33}\right) \\ +\left(\sigma_{31}\sigma_{13} - \sigma_{33}\sigma_{11} + \sigma_{32}\sigma_{23} - \sigma_{33}\sigma_{22} + \sigma_{33}\sigma_{33} - \sigma_{33}\sigma_{33}\right) \end{bmatrix} = [\sigma_{12}\sigma_{21} - \sigma_{11}\sigma_{22} + \sigma_{13}\sigma_{31} - \sigma_{11}\sigma_{33} + \sigma_{23}\sigma_{32} - \sigma_{22}\sigma$$

This is the negative of the determinate result (group each 2 terms and identify with the 3 determinates.

b) This is equal to $I_{\sigma,2}$, the second invariant of the stress tensor.

c) An invariant is a magnitude (scalar) calculated from a tensor that does not vary with rotation of the coordinate system or conversion to a different coordinate system. A vector has one invariant, the magnitude of the vector. A second order tensor has 3 invariants.

3) a) The stress tensor has 6 independent components since it is a symmetric tensor.
b) Displacement tensor, e_{ij}, has 9 independent components.
strain tensor, ε_{ij}, has 6 independent components since it is symmetric.
rotational tensor, ω_{ij}, has 3 independent components since it is antisymmetric and the cross terms are 0.
c) The generic modulus would have 6x6 or 36 components.