021206 Final Exam: Mechanics of Materials
Part 1:

1) Define the following terms:
   a) Von Misses criterion
   b) Tresca criterion
   c) Griffith Equation
   d) G_I
   e) K_I
   f) G_{Ic}
   g) K_{Ic}
   h) What are the main issues involved in adopting these terms to polymers?

2) A stress relaxation experiment in shear is used to determine G(t).
   a) What is a stress relaxation experiment and how is G(t) calculated?
   b) G(t) can serve as a "memory function" in the following equation:

$$\sigma(t) = -\int_{t^*=-\infty}^{t} G(t - t^*) \frac{d\mathbf{B}(t, t^*)}{dt^*} dt^*$$

Explain how G(t) is a memory function.

   c) Give plots of G(t) for a Hookean elastic, Newtonian fluid and a viscoelastic
   d) Plot log(viscosity) versus log(rate of strain) for a polymer fluid.
   e) Show where the limiting viscosity, η_0, is measured.
   f) How can G(t) be used to calculate this limiting viscosity?

3) Random Topics:
   a) Show how the energy absorbed by a sample in deformation can be determined from the stress strain curve.
   b) Show that the stress tensor has 6 independent components under the assumption of no rotational torque on the sample.
   c) How many independent invariants does the stress tensor have?
   d) What is the difference between true stress and engineering stress?
   e) How is this difference important to the Considere construction for necking behavior?
   f) Explain the meaning of plane stress and plane strain?

Part II

The terminal behavior (yielding and failure) for polymers and metals/ceramics have some similarities and some differences.

1) a) What two general behaviors are seen in metals and ceramics in the terminal region of the stress strain curve? (Sketch the stress strain curves and give a name for the behaviors)
b) What three general behaviors are seen for polymers? (Sketch the stress strain curves and give a name for the behavior)

2) a) Describe the morphological basis for yielding in metals. (give the name and a sketch)
b) Describe the morphological basis (model) for failure in metals. (give the name and a sketch)
c) Describe the three morphological features related to the three behaviors seen in polymers. (give the names and sketches)

3) a) Explain why BCC and FCC metals are generally ductile while ceramics and HCP metals are generally brittle at room temperature.
b) Semi-crystalline polymers always have very low symmetry crystals. Explain why semi-crystalline polymers are generally ductile.

4) a) The shear modulus of a series of polymers show a decrease with molecular weight that follows 1/M. Explain this observation.
b) Explain why a polystyrene cup turns white when it is flexed.
c) What is the Bauschinger effect? Do metals display this effect?
d) Why do metals strain harden?
e) Why might one be interested in metals with nano-size grains? (give an equation)

5) For a fiber composite give equations for
a) b) the composite moduli
c) the minimum fiber volume fraction for reinforcement and
d) the minimum fiber aspect ratio for reinforcement.

021206 Answers Final Mechanics of Materials

Part I:

1) a) Von Mises criterion is a yielding criterion based on the second invariant of the deviatory stress, yield occurs when J2 reaches a critical value,

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = k^2$$

or

$$\sigma_0 = \left(\sqrt{3}\right)k \text{ for uniaxial tension}$$

b) Tresca criterion is a yielding criterion based on the second and third invariants of the deviatory stress that says yielding will occur when a critical value is reached for the shear stress,

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_0}{2} = k$$

c) The Griffith equation is an equation that predicts the failure stress of a material and is based on the idea of native flaws in the material.

$$\sigma^* = \left(\frac{2E\gamma_s}{\pi c}\right)^{1/2}$$

d) G_I is the fracture toughness, or strain-energy release rate, for mode I fracture (tensile mode)
e) G_{Ic} is the critical value of fracture toughness, i.e. the value of G_I at fracture.

$$G_{Ic} = \frac{\pi a \sigma_c^2}{E}$$

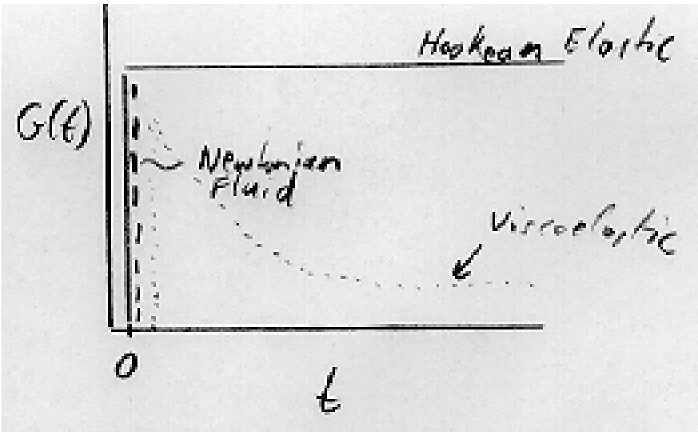
where a is half the native flaw length, σ_c is the bulk stress at failure and E is the tensile modulus.
f) K_I is the value of the stress intensity factor associated with a native flaw in the tensile mode.
g) K_{Ic} is the critical value of the stress intensity factor at failure. Using the model of a native flaw with parallel flaw lines, an infinitesimally narrow gap and a flaw tip radius of atomic dimensions,

$$K_{Ic} = \sigma(\pi a)^{1/2}$$

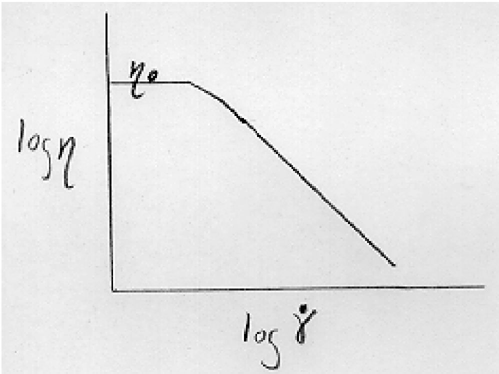
where a is half the flaw length.
h) The main issue in adopting equations a and b to polymers is that yielding in polymers is sensitive to hydrostatic pressure. The Coloumb equation is an adoption of the Tresca criterion for instance. The Griffith equation is modified for rate dependence of the crack growth using Arrhenius type functions. The fracture toughness and stress intensity factor become sensitive to rate of deformation and temperature.

2) a) In stress relaxation a fixed strain is applied to a sample (shear, tensile or other) and the stress is measured as a function of time. The shear modulus, G(t), is the shear stress divided by the fixed strain that was applied to the sample at time t=0.
b) The equation describes the "Boltzman superposition" of strains, in terms of the Finger tensor, over all of history to the present. An increment of strain applied at time t* in history is remembered in the stress now, at t, according to the materials current, time t, "memory" of events at time t*. The Boltzmann assumption is that with a memory function, G(t-t*), all strains are simply summed in a linear fashion to arrive at the current stress.

c)



d)



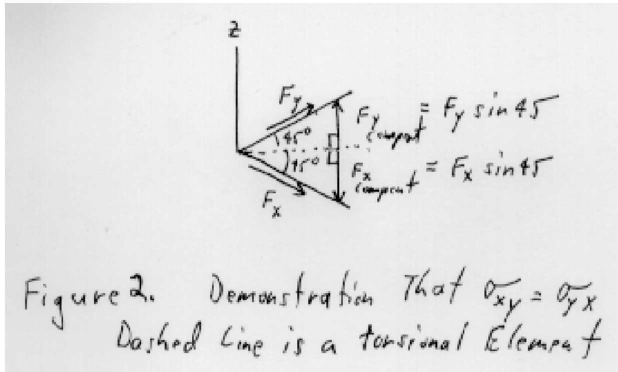
e) above

f) $\eta_0 = \int_0^\infty G(t)dt$

3)

a) The area under the stress strain curve is the energy absorbed by a sample in deformation.

b) This can be demonstrated for one pair of symmetric shear stresses, for instance σ_{xy} and σ_{yx}, below. The dashed line in figure below is a torsional bar which must have not rotational torque is the system is not subjected to rotational motion. Two stresses, σ_{xy} and σ_{yx}, are applied to the system. The torsional element is subjected to two rotational forces shown. These two forces must balance under these conditions leaving $F_y = F_x$ and $\sigma_{xy} = \sigma_{yx}$. (For each off diagonal torsional element the solid body has two symmetric torsional elements which when summed lead to the same result.)

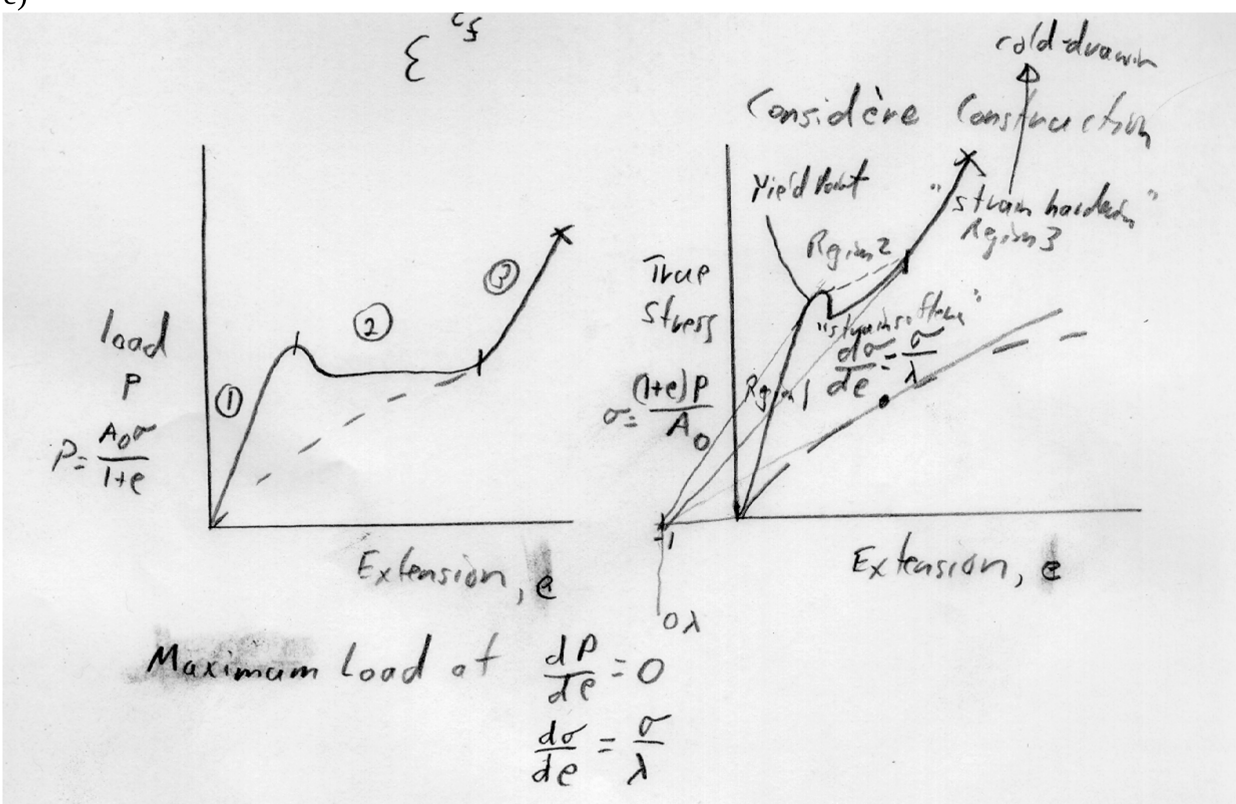


since $\sigma_{ij}=\sigma_{ji}$ there are only 6 independent components to the stress tensor.

c) A 3 x 3 tensor has 3 independent invariants.

d) True stress is the force applied to a sample divided by the actual cross sectional area of the sample. Engineering stress uses the initial area of the sample.

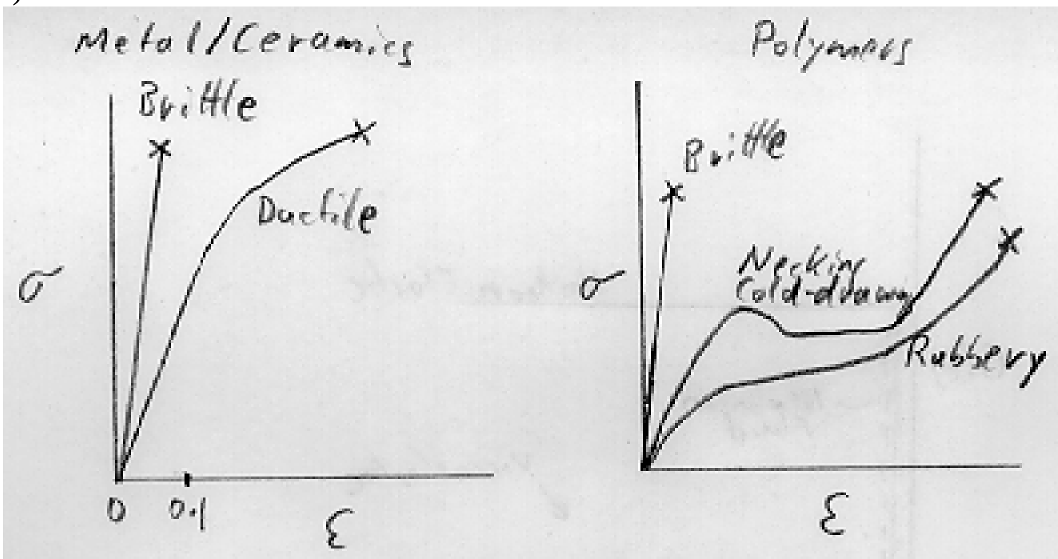
e)



f) Plane stress is expected for a thin sheet in uniaxial tension. In plane stress there are no dilatational stresses, i.e. stresses normal to the tensile direction can completely relax.
Plane strain occurs when stresses normal to the tensile direction can no relax at all such as when an infinite bulk sample is considered under uniaxial stress.

Part II:

1) a) b)



2) a) In metals the morphological basis for yielding (ductile curve above) is generally given as edge dislocations.

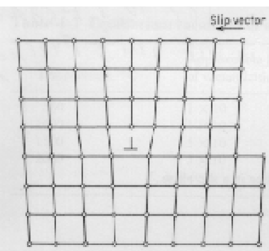


Figure 4-8 Atomic arrangement in a plane normal to an edge dislocation. *(From W. T. Read, Jr., "Dislocations in Crystals," p. 3, McGraw-Hill Book Company, New York, 1953.)*

b) Failure in metals is generally ascribed to propagation and growth of native flaws. Picture is the picture of a crack.

c) For rubber elasticity the model is a single chain being deformed

For brittle failure the morphological model is a craze which is a crack with fibrils bridging the gap. Crazes form normal to the stress direction.

For necking and cold drawing the morphological basis is shear bands which form at about 45° to the stress direction.

3) a) There are two reasons discussed in class. First pertains to the degrees of freedom for dislocation motion (the total number of slip systems) compared to the degrees of freedom for strain in an incompressible material (number of independent components which is 5). BCC has 48 slip systems, FCC has 8 and HCP has 3. So for HCP there are fewer slip directions than degrees of freedom for strain so some of the strain can not be relieved by slip of edge dislocations and the material fails in a brittle manner. This is also true for most ceramics.

The second reason has to do with the ratio between the strain field around a dislocation and the Burgess vector, w/b. A consideration of dislocation motion leads to an exponential relationship between the yield stress and this ratio, $\tau_p \approx Ge^{-k(w/b)}$.

b) Crystal deformation by edge dislocations doesn't happen in polymers. The polymer crystals are ripped apart and reform into fibrillar structures on deformation.

4) a) Rubber elasticisy, G = $\rho_c RT$
b) Crazes lead to stress whitening in polymers
c) The stress strain behavior differs mechanistically between compression and tension. Metals do not display this behavior.
d) Metals strain harden due to log jams of dislocations, sessile dislocations.
e) Hall-Petch relationship

The Hall-Petch relationship associates grain size and yield stress for a polycrystalline material, derived in the early 1950's. It has recently been used as the argument for research into nano-scale grains in polycrystalline materials.

$$\sigma_{yield} = \sigma_i + \frac{k}{D^{1/2}}$$

where σ_i is the friction stress that reflects resistance to dislocation movement in the crystal, k is the locking parameter reflecting how grains contribute to strain hardening and D is the diameter of an average grain.

5) a) $E_c = fE_f + (1-f)E_m$
b) $1/E_c = f/E_f + (1-f)/E_m$
c) $f^* = \dfrac{\sigma_{matrix,u} - \sigma_{m,uf}}{\sigma_{f,u} + \sigma_{matrix,u} - \sigma_{m,uf}}$
d) $L_{\min} = \dfrac{\sigma_{fiber} d}{2\tau_0}$